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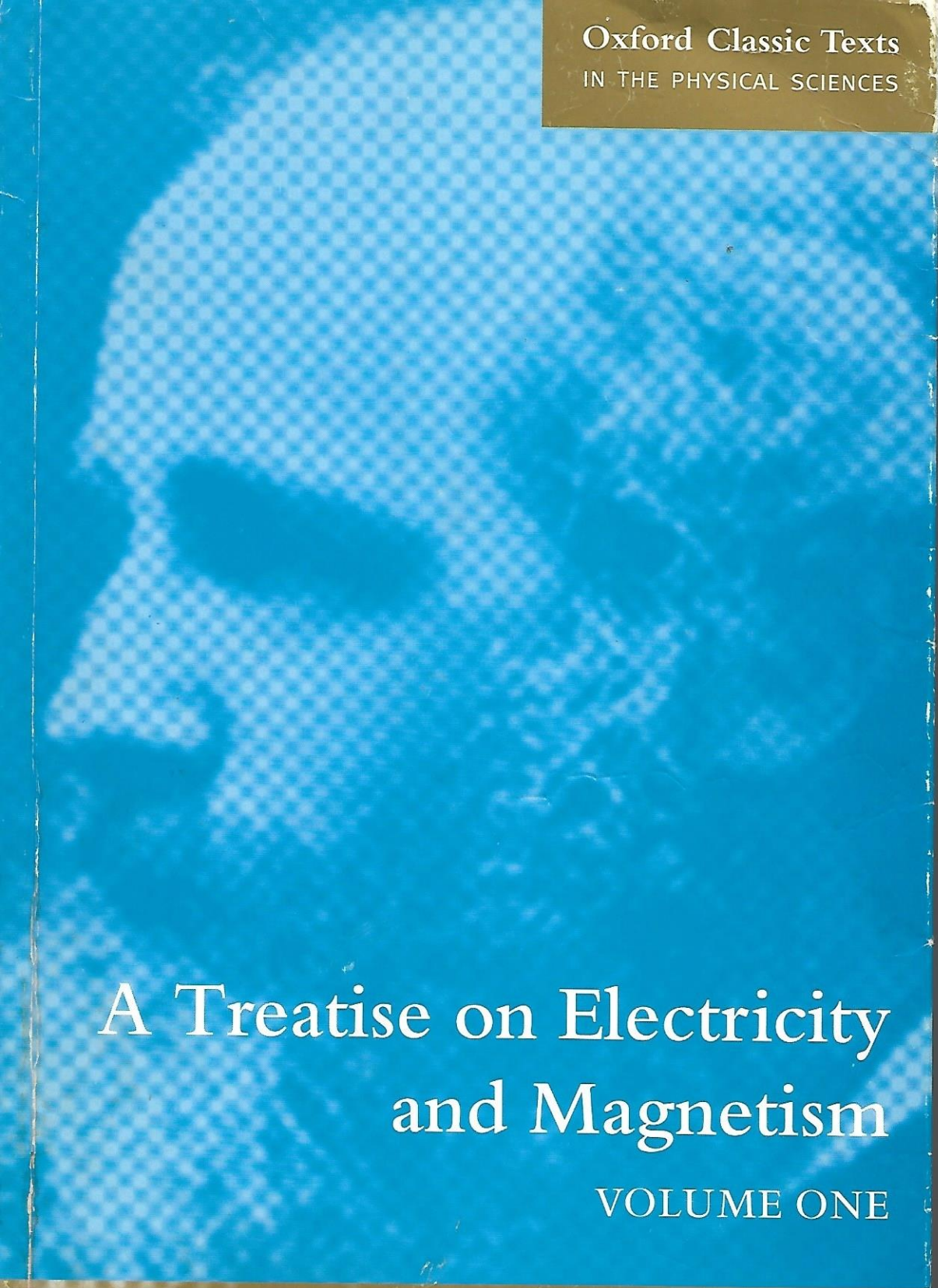
IN THE PHYSICAL SCIENCES



A Treatise on Electricity and Magnetism

VOLUME ONE

James Clerk Maxwell

$$p_4 = 5a^5b^3c^{-8} + 15a^5b^5c^{-10} + 35a^5b^7c^{-12} + (20a^8b^6 + 70a^5b^9)c^{-14}$$
$$+ (45a^{10}b^6 + 120a^8b^8 + 126a^5b^{11})c^{-16} + \ldots. \quad (10)$$

$$q_4 = a^5c^{-5} + 10a^8b^3c^{-11} + (15a^{10}b^3 + 45a^8b^5)c^{-13}$$
$$+ (20a^{12}b^3 + 90a^{10}b^5 + 140a^8b^7)c^{-15}$$
$$+ (25a^{14}b^3 + 150a^{12}b^5 + 40a^{11}b^6 + 350a^{10}b^7 + 350a^8b^9)c^{-17} +$$

$$p_5 = 6a^6b^3c^{-9} + 21a^6b^5c^{-11} + 56a^6b^7c^{-13}$$
$$+ (24a^9b^6 + 126a^6b^9)c^{-15} + \ldots. \quad (10)$$

$$q_5 = a^6c^{-6} + 12a^9b^3c^{-12} + (18a^{11}b^3 + 63a^9b^5)c^{-14}$$
$$+ (24a^{13}b^3 + 126a^{11}b^5 + 224a^9b^7)c^{-16} + \ldots. \quad (11)$$

$$p_6 = 7a^7b^3c^{-10} + 28a^7b^5c^{-12} + 84a^7b^7c^{-14} + \ldots. \quad (12)$$

$$q_6 = a^7c^{-7} + 14a^{10}b^3c^{-13} + (21a^{12}b^3 + 84a^{10}b^5)c^{-15} + \ldots \quad (13)$$

$$p_7 = 8a^8b^3c^{-11} + 36a^8b^5c^{-13} + \ldots. \quad (14)$$

$$q_7 = a^8c^{-8} + 16a^{11}b^3c^{-14} + \ldots. \quad (15)$$

$$p_8 = 9a^9b^3c^{-12} + \ldots. \quad (16)$$

$$q_8 = a^9c^{-9} + \ldots. \quad (17)$$

The values of the r's and s's may be written down by interchanging a and b in the q's and p's respectively.

If we now calculate the potentials of the two spheres in terms of these coefficients in the form

$$\alpha = lA + mB, \quad (18)$$
$$\beta = mA + nB, \quad (19)$$

then l, m, n are the coefficients of potential (Art. 87), and of the

$$m = c^{-1} + p_1ac^{-2} + p_2a^2c^{-3} + \&c., \quad (20)$$
$$n = b^{-1} - q_1ac^{-2} - q_2a^2c^{-3} - \&c., \quad (21)$$

or, expanding in terms of a, b, c,

$$m = c^{-1} + 2a^3b^3c^{-7} + 3a^3b^3(a^2 + b^2)c^{-9} + a^3b^3(4a^4 + 6a^2b^2 + 4b^4)c^{-11}$$
$$+ a^3b^3[5a^6 + 10a^4b^2 + 8a^3b^3 + 10a^2b^4 + 5b^6]c^{-13}$$
$$+ a^3b^3[6a^8 + 15a^6b^2 + 30a^5b^3 + 20a^4b^4$$
$$+ 30a^3b^5 + 15a^2b^6 + 6b^8]c^{-15}$$
$$+ a^3b^3[7a^{10} + 21a^8b^2 + 75a^7b^3 + 35a^6b^4 + 144a^5b^5$$
$$+ 35a^4b^6 + 75a^3b^7 + 21a^2b^8 + 7b^{10}]c^{-17}$$
$$+ a^3b^3[8a^{12} + 28a^{10}b^2 + 164a^9b^3 + 56a^8b^4 + 446a^7b^5 + 192a^6b^6$$

$$+ 446a^5b^7 + 56a^4b^8 + 164a^3b^9 + 28a^2b^{10} + 8b^{12}]c^{-19}$$
$$+ a^3b^3[9a^{14} + 36a^{12}b^2 + 280a^{11}b^3 + 84a^{10}b^4 + 1107a^9b^5 + 318a^8b^6$$
$$+ 1668a^7b^7 + 318a^6b^8 + 1107a^5b^9 + 84a^4b^{10} + 280a^3b^{11}$$
$$+ 36a^2b^{12} + 9b^{14}]c^{-21} + \ldots. \quad (42)$$

$$- b^{-1} + a^3c^{-4} - a^5c^{-6} - a^7c^{-8} - (a^3 + 4b^3)a^6c^{-10}$$
$$(a^5 + 12a^2b^3 + 9b^5)a^6c^{-12} - (a^7 + 25a^4b^3 + 36a^2b^5 + 16b^7)a^6c^{-14}$$
$$(a^9 + 44a^6b^3 + 96a^4b^5 + 16a^3b^6 + 80a^2b^7 + 25b^9)a^6c^{-16}$$
$$- (a^{11} + 70a^8b^3 + 210a^6b^5 + 84a^5b^6 + 260a^4b^7$$
$$+ 72a^3b^8 + 150a^2b^9 + 36b^{11})a^6c^{-18}$$
$$+ (a^{13} + 104a^{10}b^3 + 406a^8b^5 + 272a^7b^6 + 680a^6b^7 + 468a^5b^8$$
$$+ 575a^4b^9 + 209a^3b^{10} + 252a^2b^{11} + 49b^{13})a^6c^{-20}$$
$$+ (a^{15} + 147a^{12}b^3 + 720a^{10}b^5 + 693a^9b^6 + 1548a^8b^7 + 1836a^7b^8$$
$$+ 1814a^6b^9 + 1640a^5b^{10} + 1113a^4b^{11} + 488a^3b^{12}$$
$$+ 392a^2b^{13} + 64b^{15})a^6c^{-22} + \ldots. \quad (43)$$

The value of l can be obtained from that of n by interchanging a and b.

The potential energy of the system is, by Art. 87,

$$W = \tfrac{1}{2}lA^2 + mAB + \tfrac{1}{2}nB^2, \quad (44)$$

and the repulsion between the two spheres is, by Art. 93a,

$$-\frac{dW}{dc} = \tfrac{1}{2}A^2\frac{dl}{dc} + AB\frac{dm}{dc} + \tfrac{1}{2}B^2\frac{dn}{dc}. \quad (45)$$

The surface density at any point of either sphere is given by equations (1) and (4) in terms of the coefficients A_n and B_n.