

## **The Role of Displacement Current in a TEM wave**

Dear Colleagues,

there are some interesting discussions being conducted at the moment regarding the validity of Displacement current as shown in the attached extract from a book "Electromagnetic Wave Propagation" by Donald Dearholt and William R McSpadden. McGrawhill 1973.

I have a simple request for you. Can you merely comment upon whether you agree with the explanation given in the extract from the book? Thank you for your cooperation.

Best Regards,

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sending end? It would be reasonable to expect current to flow away from the battery on the conducting rod. Is this current flow continuous? For example, when the disturbance has traveled halfway down the line, how does the current get from the conducting rod to the conducting cylinder? If current flows, the battery supplies energy to the system, and some of this energy will be lost in the resistance at the receiving end. Can the transmission line store energy, and if so, how is the energy distributed along the transmission line? Rigorous answers to these questions will require the use of mathematical models; however, a brief qualitative description of the phenomena may be worthwhile as a guide through the mathematical maze.

When the switch is closed in Fig. 4.1.1, the conducting rod will be made positive with respect to the conducting cylinder because of the battery potential. Current will flow from the battery as a result of the charge redistribution, in much the same way that current flows from a battery when a capacitor is charged according to conventional circuit theory. An  $\mathbf{H}$  field will encircle the conductor carrying the current, according to Ampere's law. Similarly, an  $\mathbf{E}$  field will be established from the positive conducting rod to the negative conducting cylinder, as shown in Fig. 4.1.1b. Taking the vector product  $\mathbf{E} \times \mathbf{H}$  according to Poynting's theorem, it is seen that the Poynting vector points down the transmission line, parallel to the center conducting rod. Thus we expect power (and consequently energy) to flow into the transmission line.

From Chap. 2 recall that the derivation of the wave equation does not depend on the particular geometry in which the fields exist. Thus the electromagnetic fields within a coaxial transmission line propagate down the line according to the wave equation, and closing the switch simply establishes an electromagnetic disturbance at the sending end of the line, in much the same way that the pebble dropped in water establishes a mechanical displacement initiating wave action. The disturbance appears as a transverse electromagnetic (TEM) wave, which propagates

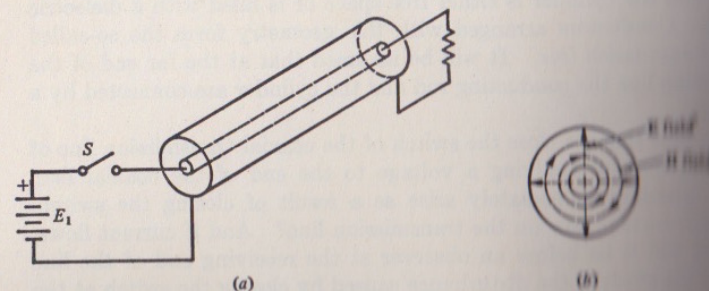


Fig. 4.1.1 The coaxial transmission line.

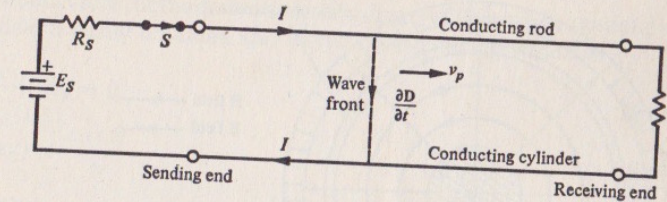


Fig. 4.1.2 Simplified drawing of the transmission line.

down the transmission line as a nonuniform plane wave. In this case the TEM wave is a step function, due to the manner in which energy was applied from the source, and an observer partway down the line would see this wavefront pass at the velocity of propagation  $v_p$ . As a result of this step function in the electromagnetic fields, an observer would detect a thin sheet of displacement-current density  $\partial\mathbf{D}/\partial t$  flowing from the conducting rod to the conducting cylinder at the wavefront. One can trace a complete path for the current starting from the battery, then down the conducting rod in the center, through the displacement current, to the outer conducting cylinder, and back to the battery. This is shown in Fig. 4.1.2, which also illustrates a simplified way of drawing two-conductor transmission lines of different geometries. Because of the dc source, the electromagnetic fields in the region between the conductors will be constant after the wavefront has passed. In the following sections, we will proceed with a mathematical analysis of transient phenomena by first deriving the relationships between voltage, current, and the fields on a transmission line.

#### 4.1 THE WAVE EQUATIONS FOR VOLTAGE AND CURRENT

The object of this section is to show that voltage and current on the transmission line are described by the wave equation. In elementary field studies it is shown that the static electromagnetic fields on a coaxial transmission line are distributed as sketched in Fig. 4.2.1. In this figure we assume a direct current into the paper in the center conductor and an equal current out of the paper in the outer conductor. Furthermore, we will assume that the conductors are perfect. The  $\mathbf{E}$  field is radial from the center conductor and has a vector component only in the  $r$  direction. A cylindrical coordinate system is assumed here, with the positive  $z$  direction into the paper.) It will be assumed that the time-varying fields have only the components  $E_r$  and  $H_\phi$ , as did the static fields, and that they can vary with both time and distance along the transmission line.†

† For further discussion of this assumption, see R. B. Adler, L. J. Chu, and R. M. M. "Electromagnetic Energy Transmission and Radiation," pp. 222, John Wiley & Sons, New York, 1954.