

# A DC Voltage is Equivalent to Two Traveling Waves on a Lossless, Nonuniform Transmission Line

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October 2012. I only just noticed "Professor" and "Fellow, IEEE." Such a High Priest could not possibly reply to suggested heresy, that the earth really *did* move, that it was not just a useful model for calculation. Note that he does not cite me, although I predate him by 13 years. Ivor Catt

**Abstract**—A static dc voltage can be treated as two traveling waves propagating in opposite directions of a lossless, nonuniform transmission line. The amplitudes of these two traveling waves are a function of the characteristic impedance of signal line. The concept of two traveling waves is applied to a time-domain-scattering-parameters analysis in a lossless, nonuniform transmission line terminated with nonlinear loads.

## I. INTRODUCTION

WHEN A LOSSLESS transmission line is connected to a dc voltage<sup>1</sup> source at one end and an appropriate resistive load at the other end, a steady state dc voltage will finally be reached in the lossless signal line. Such a dc voltage is often treated as a static signal. In this paper, we treat such a static dc signal as two traveling waves propagating in opposite directions of the lossless signal line. In general, these two traveling waves have different signal amplitudes; the summation of these two traveling waves is equal to the static dc voltage. Such an approach will give us physical insights regarding the interaction between the transmission line and associated terminations in time domain analysis [1]–[4].

## II. TRAVELING WAVE SOLUTIONS

The time-space domain solutions of a lossless, uniform transmission line are [1]

$$V(t, x) = V_+ \left( t - \frac{x}{u} \right) + V_- \left( t + \frac{x}{u} \right), \quad (1a)$$

$$I(t, x) = \frac{1}{Z} \left[ V_+ \left( t - \frac{x}{u} \right) - V_- \left( t + \frac{x}{u} \right) \right], \quad (1b)$$

where  $V$  represents voltage,  $I$  is the current,  $Z = (L/C)^{1/2}$  is the characteristic impedance,  $u = (LC)^{-1/2}$  is the wave velocity,  $L$  and  $C$  are inductance and capacitance of the signal line per unit length,  $t$  is the time and  $x$  is the space variable. Note that  $V_+(t - x/u)$  and  $V_-(t + x/u)$  represent the waves traveling in the  $+x$  (forward) and  $-x$  (backward) directions, respectively.

We assume that a uniform transmission line having a characteristic impedance  $Z$  is loaded with a dc voltage  $V_S$ , source resistance  $R_S$  at the left-hand side and a dc voltage  $V_L$ , resistor load  $R_L$  at the right-hand side. For such a

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<sup>1</sup>The terminology "dc voltage" here and in the title might indicate not a continuous dc voltage but a long-period pulse.

to the circuit configuration at the load (right) end. For the circuit previously stated, the traveling wave  $V_{x+}(t, l)$  makes a contribution to the total incident and reflected waves  $a_2(t)$ ,  $b_2(t)$  at the load (left) end and the existence of  $V_{x+}(t, l)$  extends over the time interval  $t_l < t < \tau + t_s$ . Notice that the summation of existing time of traveling waves  $V_{x+,-}$  at both ends of the line is  $2\tau$ .

#### IV. CONCLUSION

We decomposed a dc voltage on a lossless, nonuniform transmission line into two traveling waves propagating in opposite directions of the signal line. The amplitudes of two traveling waves are symmetric with respect to a horizontal line representing half of the steady state voltage. This approach

provides us physical insights regarding the interaction between transmission lines and associated loads in time-domain considerations.

#### REFERENCES

- [1] R. B. Adler, L. J. Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation*. New York: Wiley, 1960.
- [2] F. H. Branin, "Transient analysis of lossless transmission lines," *Proc. IEEE*, vol. 55, pp. 2012-2013, Nov. 1967.
- [3] C. T. Tai, "Transients on lossless terminated transmission lines," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 556-561, July 1978.
- [4] C.-W. Hsue, "Elimination of ringing signals for a lossless, multiple-section transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1178-1183, Aug. 1989.
- [5] J. E. Schutt-Aine and R. Mittra, "Scattering parameter transient analysis of transmission lines loaded with nonlinear terminations," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 529-536, Mar. 1988.