Oliver Heaviside, "Electromagnetic Theory" volume 3. Pub. 1900-1911. Previous volumes began 1891. Previously published were his 2 vol. "Electrical Papers", pub. 1872-1892 See http://www.ivorcatt.co.uk/2604.htm

CHAPTER IX.

WAVES FROM MOVING SOURCES.

1 Adagio. Andante. Allegro moderato.

§ **450.** The following story is true. There was a little boy, and his father said, "Do try to be like other people. Don't frown." And he tried and tried, but could not. So his father beat him with a strap; and then he was eaten up by lions.

Reader, if young, take warning by his sad life and death. For though it may be an honour to be different from other people, if Carlyle's dictum about the 30 millions be still true, yet other people do not like it. So, if you are different, you had better hide it, and pretend to be solemn and wooden-headed. Until you make your fortune. For most wooden-headed people worship money; and, really, I do not see what else they can do. In particular, if you are going to write a book, remember the wooden-headed. So be rigorous; that will cover a multitude of sins. And do not frown.

There is a time for all things: for shouting, for gentle speaking, for silence; for the washing of pots and the writing of books. Let now the pots go black, and set to work. It is hard to make a beginning, but it must be done.

Electric and magnetic force. May they live for ever, and never be forgot, if only to remind us that the science of electromagnetics, in spite of the abstract nature of the theory, involving quantities whose nature is entirely unknown at present, is really and truly founded upon the observation of real Newtonian forces, electric and magnetic respectively. I
2 cannot appreciate much the objection that they are *not* forces; because they *are* the forces per unit electric and magnetic pole. All the same, however, I think Dr. Fleming's recent proposal that electric force and magnetic force shall be called the voltivity and the gaussivity a very good one; not as substitutes for with abolition of the old terms, but as alternatives; and beg to recommend their use if found useful, even though I see no reason for giving up my own use of electric and magnetic force until they become too antiquated.

Having thus got to the electric and magnetic forces, it is only a short step farther to near the end of the book—namely, to the simple cases in which they occur simultaneously. It does not follow that the matter which comes towards the end of a treatise—for instance, Maxwell's great work—is harder than that in the first chapter of his Vol. I. On the contrary, some parts of it are easier out of all comparison. In the course of the next generation many treatises on electromagnetics will probably be written; and there is no reason whatever (and much good reason against it) why the old-fashioned way of beginning with electrostatics (unrelated to the general theory) should be followed. After all, should not the easier parts of a subject come first, to help the reader and widen his mind? I think it would be perfectly practical to begin the serious development of the theory with electromagnetic waves of the easy kind. First of all, of course, there should be a good experimental knowledge all round, not necessarily very deep. Then, considering the structure of a purely theoretical work to co-ordinate the previous, a general survey is good to begin with, with consideration in more detail of the properties of circuits and the circuital laws. Then, coming to developments, start with plane electromagnetic waves in a dielectric non-conductor. The algebra thereof, even when pursued into the details of reflections, &c., is perhaps more simple than in any other part of the science, save Ohm's law and similar things; and the physical interest is immense. You can then pass to waves along wires. First the distortionless theory in detail, and then make use of it to establish the general nature of the effects produced by practical departures from such perfection, leaving the difficult mathematics of the exact results for later treatment. Now, all this and much
3 more is ever so much easier than the potential functions and spherical harmonics and conjugate transformations with which electrostatics is loaded, and there is more exercise for the brains in the electromagnetic than in the electrostatic problems. The subsequent course may be left open. There are all sorts of ways.

Simple Proof of Fundamental Property of a Plane Wave.

§ 451. At present, in dealing with some elementary properties, the object is to smooth the road to the later matter. First of all, how prove the fundamental property of a plane wave, that it travels at constant speed undistorted, if there be no conductivity, or, more generally, no molecular interference causing dispersion and other disturbances? We have merely

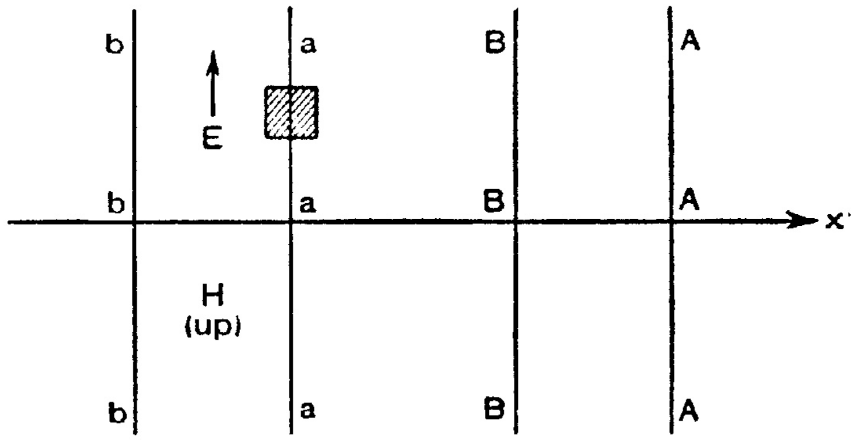

Fig. 1.

to show that the two circuital laws are satisfied, and that can be done almost by inspection. Thus, let the region between two parallel planes *aaa* and *bbb* be an electric field and a magnetic field at the same time, the electric force **E** being uniform and in (and parallel to) the plane of the paper, whilst **H** is also uniform, perpendicular to **E** and directed up through the paper. Also let their intensities be connected by $\mathrm{E}=\mu v\mathrm{H}$, or $\mathrm{H}=cv\mathrm{E}$; c being the permittivity and μ the inductivity, whilst v is defined by $\mu cv^2=1$. This being the state at a given moment, such as would be maintained stationary by steadily acting impressed forces $\mathrm{e}=\mathbf{E}$ and $\mathrm{h}=\mathbf{H}$ in the slab between the planes, what will happen later, in the absence of all impressed force?

Apply the two circuital laws. They are obviously satisfied for all circuits which are wholly between the planes a and b, or else wholly beyond them to right or left. There are left only circuits which are partly inside and partly outside the slab. Consider a unit square circuit in the plane of the paper, as shown in the figure. The circuitation of **E** is simply E, and by the second circuital law this must be the rate of decrease of induction $\mathbf{B}=\mu\mathrm{H}$ through the circuit downward, or its rate of increase upward. Then turn the square circuit at right angles to the paper. The circuitation of H is then simply H, and by the first circuital law this must be the rate of increase of displacement $\mathbf{D}=\mathbf{E}c$ through the circuit. Now let the plane aaa move to the right at speed v. The two rates of increase are made to be $v\mu\mathrm{H}$ and $vc\mathrm{E}$ respectively. That is, $\mathrm{E}=\mu v\mathrm{H}$ and $\mathrm{H}=cv\mathrm{E}$ express the circuital laws. They are harmonised by the definition of v. We prove that the circuital laws are satisfied in the above way. That there is no other way of putting induction and displacement in the two circuits may be seen by considering circuits two of whose sides are infinitely near the plane a on opposite sides. The fluxes must be added on just at the plane itself, extending the region occupied by **E** and **H**. Similar reasoning applied to the plane bbb proves that it must also move to the right at speed v. Thus the whole slab moves bodily to the right at speed v, so that a moves to A and b moves to B in the time given by $vt=a\mathrm{A}$ or $b\mathrm{B}$.

The disturbance transferred in this way constitutes a pure wave. It carries all its properties with it unchanged. The density of the electric energy, or $\mathrm{U}=\frac{1}{2}c\mathrm{E}^2$, equals the density of the magnetic energy, or $\mathrm{T}=\frac{1}{2}\mu\mathrm{H}^2$. The flux of energy is $\mathbf{W}=\mathbf{v}(\mathrm{U}+\mathrm{T})$, the simplest case of the general formula $\mathbf{W}=\mathrm{V}(\mathbf{E}-\mathbf{e})(\mathbf{H}-\mathbf{h})$. *See* Vol. I., § 70.

The General Plane Wave.

§ **452.** What is proved for a discontinuity is proved for any sort of variation. For the slab may be of any depth and any strength, and there may be any number of slabs side by side behaving in the same way, all moving along independently and unchanged. So $\mathrm{E}=\mu v\mathrm{H}$ expresses the general solitary wave, where, at a given moment, E may be an arbitrary function of x, real and single-valued of course, but without any necessary continuity in itself or in any of its differential

coefficients. Denoting it by $f(x)$ when $t=0$, it becomes
5 $f(x-vt)$ at the time t.

If we change the sign, and make $\mathrm{E} = -\mu v\mathrm{H}$, this will represent a negative wave, going from right to left. There may be a positive and a negative wave coexistent, separate in position, or superimposed. This constitutes the complete solution for plane waves with straight lines of **E** and H. If E_0 and H_0 are given arbitrarily (with no connection) at the moment $t=0$, the two waves at that moment are

$$\text{(positive)} \quad \mathrm{E}_1 = +\mu v\mathrm{H}_1 = \tfrac{1}{2}(\mathrm{E}_0 + \mu v\mathrm{H}_0),$$
$$\text{(negative)} \quad \mathrm{E}_2 = -\mu v\mathrm{H}_2 = \tfrac{1}{2}(\mathrm{E}_0 - \mu v\mathrm{H}_0),$$

as may be immediately verified. Move E_1 to the right, and E_2 to the left, at speed v, to produce the later states.

Since every slab is independent of the rest, there need be no connection between the directions of E in one slab and the next. The direction may vary anyhow along the wave. This makes a mathematical complication of no present importance, the behaviour of individual slabs being always the same.

The overlapping of positive and negative waves should be studied to illustrate the conversion of electric to magnetic energy, or conversely. For two equal waves moving oppositely, which fit when they coincide, there is a complete temporary disappearance and conversion of one or the other energy, according as **E** is doubled, leaving no H, or H is doubled, leaving no E. If $\mathbf{E}_0$ exists alone initially, it makes two equal oppositely-going waves, of half strength as regards **E**. Similarly as regards initial H_0.

The reversal of sign of both **E** and **H** in either a positive or a negative wave does not affect the direction of motion. But if only one be reversed, it is turned from a positive to a negative wave, or conversely. Slabs of uniform strength should be studied, not simply periodic trains of waves, for simplicity of ideas. Only when there is dispersion, and the wave speed varies, is it necessary to consider a train of waves of given frequency or given wave-length; because then a slab spreads out behind as it travels, producing a diffused wave of difficult mathematical representation, by reason of the partial reflection of its different parts as it progresses.

Generation of Waves by a Plane Source of Induction.

§ 453. Consider next how to generate plane waves by impressed electric or magnetic force. Say by **e** first. It must obviously be of the same type as **E**, *i.e.*, in uniform slabs. But it is not **e** itself, but its curl, say **f**, that is the real source of the waves. A plane surface of **f** is the simplest case. Let **e** be uniform on the right side and zero on the left side of the plane AA, beginning to act at the moment $t=0$, and continuing steady later. What will happen? Here

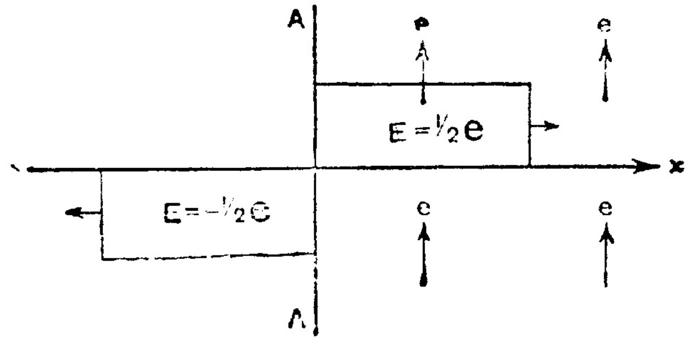

Fig. 2.

f the curl of **e** is uniform on the sheet AA; its density is $f=e$, and its direction is upward through the paper. It generates induction at the rate f on the plane AA per unit area. Or we may say that the strength of the source of B is f. Once generated, the induction divides fairly to right and left, and since the speed is v, the amount $\frac{1}{2}f$ is spread over the distance v in a second. Therefore

$$B=\frac{f}{2v}, \qquad \text{or} \qquad H=\frac{f}{2\mu v}=\pm cvE \tag{1}$$

expresses the full connection between f and the waves generated. The plus sign is for the positive wave, and the minus sign for the negative wave. We introduce E in this way because the disturbance, once started, makes free waves. The solution, of course, only holds good up to the two wave fronts, which are at distance vt from the source. **H** is up through the paper in both waves. Here we see the inner meaning of the impedance $2\mu v$ of the doubly infinite unit tube of flux of energy. It depends essentially upon the speed with which the medium can carry away from the source the induction supplied there. The value of **E** is $\frac{1}{2}e$ on the right and $-\frac{1}{2}e$ on the left side of the plane source. Since **E** is the force of the flux **D**, the force of the field, or **E** $-$ e, is of value $-\frac{1}{2}e$ all the way between the two wave fronts.

When f varies anyhow in time, it is just the same as regards the generation of **B**. If f is impulsive, it makes two impulsive waves. Equations (1) may be used when f is variable, if we understand that the elementary slab of **E** and **H** referred to, say at distance x, belongs to the f at the source at the moment earlier by the amount x/v.

It is e that does the work, though, but only where there is electric current. That is, only at the wave front on the positive side, when the source is steady, and intermediately only when f varies; because $-d\mathrm{H}/dx$ measures the current density. The flux of energy **W** is V(**E** $-$ e)H. It is entirely from the right to the left wave front when f is steady. It may be tested that

$$\mathrm{e}\dot{\mathbf{D}} = \dot{\mathrm{U}} + \dot{\mathrm{T}} + \mathrm{div}\,\mathbf{W} \tag{2}$$

is the equation of activity in any case.

In the case of a uniform slab of e of finite depth, there are two plane sources of f, acting oppositely, or one as a source in the above manner and the other simultaneously as an equal sink of induction. There are therefore two positive and two negative waves, separate up to a certain time, and then superimposed. Four stages are shown in the figure.

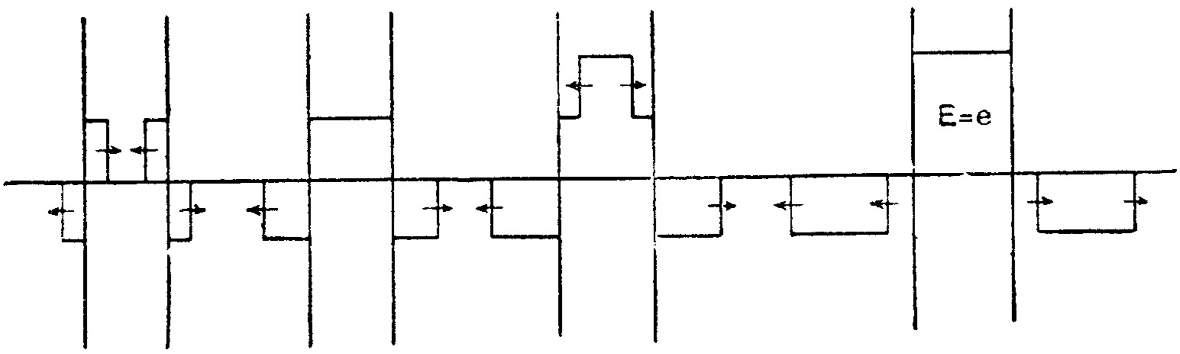

FIG. 3.

First, before the initial waves have begun to overlap inwardly; secondly, when overlapping has just commenced; thirdly, before the overlapping is completed; fourthly, a little while after completion of overlapping, showing the emergence of two pure waves. It is the **E** that is referred to. Every wave has its corresponding **H** according to $\mathrm{E} = \pm \mu v \mathrm{H}$. The final result is $\mathrm{E} = e$ in the slab, and two pure waves in 8
which $\mathrm{E} = -\frac{1}{2}e$. The energy of these free waves together equals that of the steady flux of **D** without **B** which is established in the slab. The work is done by **e** where **e** and **D** coexist, that is, at the wave fronts in the region of **e**. The total induction is zero, because there are two opposite **f**'s. The total displacement is also zero, for another reason.

Generation of Waves by a Plane Source of Displacement.

§ 454. In order to generate displacement finitely in total amount, we require another kind of source. Let the impressed force be magnetic, say **h**. Let its negative curl be **g**. Then **g** generates **D** exactly in the same way as **f** generates **B**, as before described in detail. Thus, considering a single plane source, g is the total displacement generated per unit time per unit area of the plane, and $\frac{1}{2}g$ the amount going each way, spreading over the distance v in a second. The result is that a steady source makes

$$\mathrm{D} = \frac{g}{2v}, \quad \text{or} \quad \mathrm{E} = \frac{g}{2cv} = \pm \mu v \mathrm{H} \tag{3}$$

between the plane of **g** and the two wave fronts. The same Fig. 2 will do for this case, only for E must be understood H in the two waves. Otherwise stated, $\mathrm{H} = \frac{1}{2}h$ on the right side and $-\frac{1}{2}h$ on the left; whilst $\mathrm{H} - h = -\frac{1}{2}h$ on both sides. The work is done by **h** only where there is magnetic current.

If the impressed force in the slab is **e**, the problem represented is that of the effect of suddenly electrising the slab intrinsically. If it is **h**, it means that the slab is suddenly magnetised intrinsically to density of magnetisation $\mathrm{I} = \mu \mathrm{H} = \mu h = \mathrm{B}$. This represents the complete and full induction possible. If it is a material slab, and the inductivity differs from that outside, the waves will not be quite so simple. But it is only when the slab is of finite depth that we can get the full induction; when of infinite depth one way, the final result is only half as much; no steady state is reached; the flux of energy continues indefinitely. And if the slab is infinitely extended both ways, a uniform **h** cannot produce any induction in it. There is no **g** source.

End of section

at speed v. Thus the whole slab moves bodily to the right at speed v, so that a moves to A and b moves to B in the time given by $vt = a\mathrm{A}$ or $b\mathrm{B}$.

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Here he promotes "The Rolling Wave"

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