The history of displacement current

Further explanation of an earlier article

by **I. Catt** and **M. F. Davidson** (CAM Consultants) and **D. S. Walton** (Icthus Instruments Ltd)

As a result of correspondence following their article "Displacement current" in the December 1978 issue, the authors feel that further explanation of their views is required. They offer it in the form of this brief historical survey.

IN THE EARLY nineteenth century electromagnetic theory made advances, a cornerstone of the theory being the doctrine of conservation of charge q, which developed into the doctrine of continuity of electric current flow, $dq/dt = i$.

In the middle of that century Maxwell struggled with the paradox of the capacitor, where charge entered one plate and then flowed out of the other plate apparently without traversing the space between the plates (Fig. 1). It seemed that electric charge was being destroyed on the upper plate and being re-created when it reappeared on the lower plate. Maxwell "cut the Gordian knot" as Heaviside put it (Heaviside 1893) by postulating a new type of current, called "displacement current", as flowing across the gap BC in Fig. 1 so as to save the principle of continuity of electric current.

"Displacement current" was a result of his postulation of "electric displacement". Maxwell said that the total outward displacement across any closed surface is equal to the total charge inside the closed surface (Maxwell 1873).

C.A.M.

It is not surprising that objections were raised. Notice, in Fig. 2, that if in any circuit there should be a break, BC, in the current path, we are bound by the principle of conservation of charge to say that the current i, that is the flow of charge, entering B from A accumulates as charge $\int i dt$ at B, and the current reappearing at C "accumulates" as equal negative charge $-\int i dt$. By definition, electric displacement outward from B equals the total charge trapped at B; $D = \int i dt$ and $i = dD/dt$. It is not a coincidence that "displacement current" saves the idea of continuity of electric current; *it does so by definition.* With the postulation of displacement current, it would never in future be possible to devise an experiment which might refute the principle of continuity of electric current. Popper would therefore say that "displacement current" is an unscientific concept (Popper 1963). Whenever charge seems to disappear at a point, displacement takes its place. Whenever electric current seems to disappear at a point, displacement current takes its place.

It is important that Maxwell and Heaviside believed that the current entering a capacitor plate became trapped and had nowhere to go. Writers on the subject must be glad that some route between B and C for real current did not declare itself, since they say that the brilliant postulation of displacement current led to the postulation by Maxwell of waves in space.

Meanwhile, even as Maxwell was contemplating the ethereal displacement current, practical electricians were inventing and building wired telegraph systems. The distortion of signals travelling long distances was bad, and was thought to be due to the fact that the capacitance of the telegraph wires had to be charged up through the resistance of the wires, resulting in an RC time constant which attenuated different frequencies differently. As late as 1910 virtually all electricians (including Lord Kelvin) did not accept Oliver Heaviside's claim that a telegraph wire had distributed inductance as well as capacitance, and that if only this inductance were increased by the addition of periodic loading coils, distortion-free transmission over long distances could be achieved (Heaviside 1893).

It was important for Heaviside to encourage a sensible approach to the characteristic impedance of telegraph lines, because the practical pay-off in telegraphy and telephony would be immense. (This misunderstanding delayed the introduction of telephones for twenty years.) This practical pay-off would be best achieved by arguing that signals travelling down (between) telegraph lines were undistorted TEM and similar to the waves in space discovered by Hertz in 1887, twenty years before, and previously postulated by Maxwell as one implication of his proposed displacement current.

It was important for Heaviside not to criticise the theory he was trying to argue from, Maxwell's electromagnetic theory. So it would have been injudicious for Heaviside to question the concept of displacement current, and he never did.

The essence of the concept of a transverse electromagnetic wave, TEM, is that nothing – field, flux, or current – flows laterally across the surface of the wave front. The analogy is the Severn Bore, where we see a single step of water rushing up the River Severn. Everything ahead of the step is steady, and everything behind the step is steady. There is no lateral, sideways flow. In the electromagnetic case (Fig. 3), the idea of a lateral flow of current

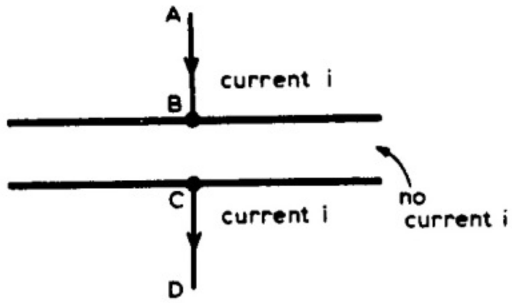

Fig. 1. *Charge flowing into one plate of a capacitor, as current i, and flowing out of the other plate.*

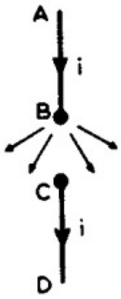

Fig. 2. *Electrical circuit AD with a break in the current path at BC. Charges accumulate at B and C.*

across the face of a TEM step is absurd, and would result in a longitudinal magnetic field; the step would "get ahead of itself". Further, since the step travels forward at the speed of light, $1/\sqrt{\mu\epsilon}$ any lateral flow would cause embarrassment by travelling even faster, in the same way that when you walk across inside a moving train by Pythagoras' Theorem you are travelling faster than the train.

Now although in the case of a capacitor, displacement current needed to be regarded as just like a real current, for instance causing a magnetic field; in the case of the D flux at the front of a step of TEM $(E \times H)$ energy current travelling down a telegraph line, the displacement clearly must not behave like a real current – for instance by creating a magnetic field which would reach out ahead of the wave front and ruin its TEM nature.

Maxwell and later Heaviside did not notice the discrepancy in the requirements of displacement current; that in a capacitor it must act like a real current but in a transmission line it must not; because neither of them knew that a capacitor is no more nor less than a transmission line (*Wireless World*, Dec. 1978, p. 51). This is even today known by very few scientists. Maxwell, along with today's text-book writers (e.g. Fewkes 1956, Bleaney 1957), believed that the displacement current dD/dt travelling across between the plates of a capacitor BC was uniformly distributed, and it is only very recently that it has been pointed out that the flow of current and field in a capacitor is identical with that in a transmission line; that the field moves out from the capacitor's leads as if they were links to one end of a transmission line. So the discrepancy could not become apparent.

A serious difficulty for displacement current arises when we realize that the two plates, BB′, CC′ in Fig. 4, *are* a transmission line. We know that the current i travelling down to B from A then flows out sideways from B along the capacitor plate BB′. This route, along the capacitor plates, failed to declare itself to Maxwell, and everyone has followed his lead.

In a transmission line (Fig. 4), everyone agrees that the current i entering the line at B leaves B by flowing along the line BB′. No displacement current dD/dt between the lines is needed for us to retain the doctrine of conservation of charge and conservation of current. In fact, if this dD/dt *were* regarded as a current, far from saving the doctrine, it would destroy it, because now more current $(i + dD/dt)$ would be leaving the first section of the plate BB′ than was entering it. The last sentence is difficult to grasp; no matter, because it is easy to see, and sufficient to see, that if i enters B from A and i leaves B along BB′, continuity of current is preserved without our having to postulate displacement current.

"But surely we cannot just drop displacement current when for a century every expert (e.g. Solymar 1976, Winch 1963) has been protesting that it is the foundation of our craft; that 'Maxwell's leap of genius' in proposing displacement current was what got the subject going – leading to Hertz's discovery of waves in space, for instance?"

The answer lies hidden in Heaviside's magnificent, regal statement, "We reverse this." In his "Electrical Papers", Vol. 1, 1892, page 438, Heaviside wrote;

> Now, in Maxwell's theory there is the potential energy of the displacement

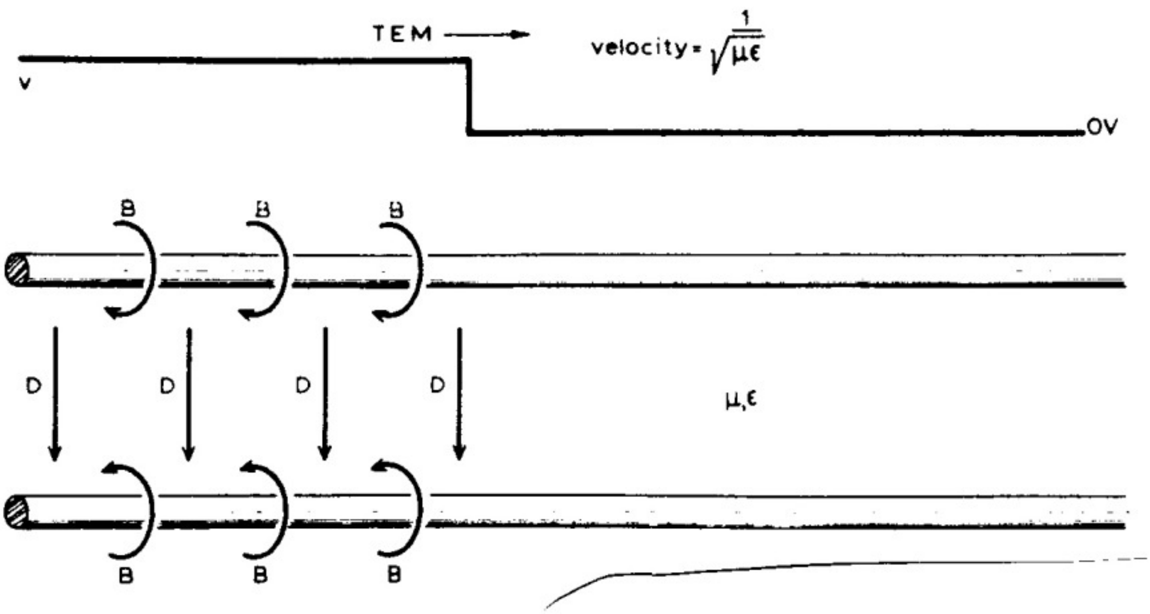

Fig. 3. *A TEM step (top) travelling at the speed of light and guided by two wires (below). The B arrows represent magnetic flux lines and the D arrows electric strain between the wires.*

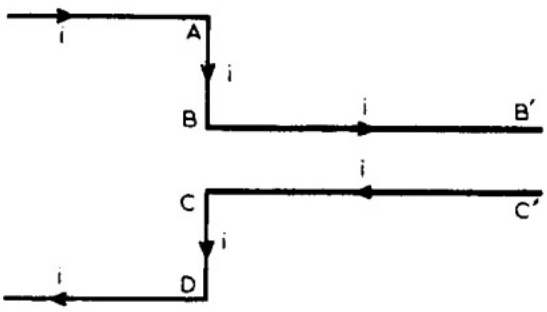

Fig. 4. *Current flowing into and out of capacitor plates BB′ and CC′. These two plates together constitute a transmission line.*

produced in the dielectric parts by the electric force, and there is the kinetic or magnetic energy of the magnetic induction due to the magnetic force in all parts of the field, including the conducting parts. They are supposed to be set up by the current in the wire. We reverse this; the current in the wire is set up by the energy transmitted through the medium around it . . .

The discrediting of displacement current merely makes Heaviside's "We reverse this" mandatory. It means that the field must be the cause and electric current an effect, rather than (as Maxwell thought) the other way round.

If we keep to "Theory H", the theory that the field $E \times H$, travelling along between the wires at the speed of light – what Heaviside called the "energy current", is the cause, then electric charge and electric current are merely what define the *edge* of an energy current. If electric current is that which defines the de of an energy current, then we may with equal justification postulate "displacement current" as that which defines the front face of a step of energy current. Under "Theory H", Maxwell's 'leap of genius' (in postulating displacement current and thence waves in space) becomes tautological; "Because a wave in space if it existed would have to have a front face (displacement current), then I propose such a front face and therefore I propose waves in space."

Maxwell would have saved us a century of confusion if he had had enough insight to say, "Since circuits containing capacitors, that is, open circuits, work, it follows that the essence of electromagnetics cannot be electric current in closed circuits of conductors; it must be something else. What about waves in space?" Heaviside, seventy years ago, missed the key point by a whisker. He failed, but he failed gloriously. He never discovered the flaw in the structure, displacement current.

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Further reading

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WIRELESS WORLD, MARCH 1979

DISPLACEMENT CURRENT

I am slightly alarmed by some of the statements in the article "Displacement current — and how to get rid of it" (December 1978). I suggest that there would justifiably be an outcry if the authors were to have written paragraph 5 as follows . . .

Since the inductance has now become a transmission line, it is no more necessary to postulate 'magnetic flux' in an inductor than it is necessary to do so for a transmission line. The excision of 'magnetic flux' from electromagnetic theory has been based on arguments independent of the classical dispute . . . (an apparent negation of Faraday's law of Induction).

Displacement current (without the inverted commas) is as real and justifiable a concept as conduction, or convection, current in charge transport — it is directly analogous to the time differential of magnetic flux in magnetic theory ($\partial\bar{D}/\partial t$ instead of $\partial\bar{B}/\partial t$ if you want to be precise). Displacement current is neither a mathematical convenience nor an artefact of a faulty model for a capacitor, it is a fundamental part of Maxwell's equations.

To those who have designed high frequency networks, interchanging between a capacitor or inductor and a transmission line is common practice: the inductors and capacitors used actually look like short transmission lines. Such circuits can be analysed using either of two methods; the discrete approach in which case each line has an equivalent inductance and capacitance or the distributed approach in which case characteristic and terminating impedances are important. Paragraph 4 could be misleading because it confuses the lumped and distributed techniques: a transmission line used as a capacitor, or a capacitor appearing as a transmission line, must have some inductance which is inherent in the component construction. This will become clear in the next paragraph.

Consider an ideal transmission line. For analysis this has a few useful parameters; L – the series inductance per unit length, C – the shunt capacitance per unit length, Z_o – the characteristic impedance ($=\sqrt{L/C}$), and v – the characteristic velocity ($1/\sqrt{LC}$). (And where do we get these parameters from? Why, of course, from electromagnetic theory using $\overline{B},\overline{H},\overline{E},\overline{J}$, and naturally enough $\overline{D}$ the electric flux or displacement vector.) The impedance measured at the end of an open circuited transmission line of length d is simply $Z_{in}=Z_o/j\tan(\omega d/v)$. But if $(\omega d/v)$ is small, a condition of lumped circuit analysis, we can expand the tan term to obtain $Z_{in} = Z_o/(j\omega d/v) + \frac{1}{3}Z_o(j\omega d/v)$. Using the transmission line parameters this gives $Z_{in} = 1/j\omega(dC) + j\omega(dL/3)$ which can be interpreted quite easily as a capacitor and inductor in series. To me that would seem a very plausible mechanism for an internal series inductor in a capacitor.

At 'low frequencies' a capacitor may well be a good equivalent circuit for a particular form of transmission line, but at increased frequencies the series inductance must be considered: eventually we must switch to a distributed analysis, otherwise we are going to be barking up the wrong tree in the wrong ball park. For digital systems where harmonics extend into the GHz region very careful consideration must be given to distributed effects in what are nominally lumped components.

P. I. Day
Maidstone
Kent

The authors reply:

We would like to make three points which we hope will clear up any misunderstanding that Mr Day has over the statements we made.

1. He wrongly assumes that we say inductance does not exist. Series inductance does not exist as a separate entity, but distributed inductance does, linked to distributed capacitance as a measured property of a transmission line defined as characteristic impedance.
2. We are considering an ideal step response of a component and the inclusion of frequency in the discussion is making an unnecessary complication.[1].
3. If Mr Day believes that you can swap "magnetic flux" with the displacement vector (current) then where does this exist when a step is propagating down a transmission line?

I. Catt, M. F. Davidson and D. S. Walton

Reference

1. Interconnection of logic elements, *Wireless World* June 1978, p. 61.

WIRELESS WORLD, APRIL 1979

COMPUTER BUSES

Ian Witten's article on computer buses (February issue) is right to point out certain key factors when designing a bus system. For example, t.t.l. totem pole gates cannot be connected together in a wired AND function for the precise reasons he states.

Then he discusses two alternatives, tristate and open collector. He dismisses the latter because of alleged slowness. This is not true. Any switching edge which appears slow is due to the quality of the interconnection, it is not an inherent property of the transistor circuit. I have achieved switching speeds of about 6-8ns, with the positive edge being slightly faster than the negative. Additionally, crosstalk calculations need to take into account the transmitting element, the receiving element and the interconnections.

element, the receiving element and the interconnections.

The possibility of excessive current as shown in the totem pole configuration also applies to tri-state if two enables happen to be true at the same time.

For a comprehensive explanation of open collector bus driving see the June 1978 issue of *Wireless World*, p. 61.

M. F. Davidson
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St Albans
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The author replies:
Malcolm Davidson raises the interesting question, to what extent should a bus be treated as a transmission line? If really high switching speeds are required, then of course the transmission line approach is mandatory, and if the line is properly terminated at both ends, open-collector gates can indeed be fast. However, one great attraction of a bus structure for interconnection of computer subsystems is the flexibility it offers for reconfiguration by inserting or removing modules, or extending the bus without adverse effect on the rest of the system – and this rules out the possibility of exactly matched termination. Fortunately, most commercial microcomputers are slow enough to allow one to get away with this! Under conditions of incorrect line termination tri-state devices give more robust and reliable performance than open-collector.

He is quite right to point out that two tri-state gates driving the same bus line should not be enabled at the same time. This means that some lines cannot be tri-state driven. The "bus request" signal illustrates this – one cannot guarantee that bus requests will not occur simultaneously. In practice, computer buses usually have some tri-state and some open-collector lines, tri-state being used when the protocol guarantees that no two devices can simultaneously drive the line. This means, of course, that failure of a device to observe the protocol may result not only in logical breakdown of communication along the bus, but in physical breakdown as well – due to multiple gates driving the line. As Mr Davidson notes in the article he cites, this is a considerable disadvantage of tri-state driving.

Ian H. Witten

DISPLACEMENT CURRENT

I would be grateful, sir, if you would kindly give me some space to point out, and endeavour to correct, certain errors and misconceptions that occur in the article "Displacement current" by Catt, Davidson and Walton in the December 1978 issue.

To say, as they do, that inductance does not exist in a capacitor is just not true: it may be small, but nevertheless it is there. A bifilar resistor has inductance but it is made small by doubling the wire back on itself in the form of a hair pin so as to give a loop enclosing a small area. The same is true when the wire is replaced by a thin conducting sheet doubled back on itself. Snipping the sheet along the folded edge gives, when rolled up, a rolled-foil capacitor – but it still has inductance. In field terms, this inductance represents the magnetic field in the very narrow space occupied by the dielectric. It is quite valid, as the authors do, to consider the capacitor as a transmission line; indeed it is necessary to do so if the length of the foils (or the radius of the circular plates in the authors' Fig. 1) is comparable with a wavelength. In undertaking such an analysis, it is necessary to consider the inductance and capacitance (respectively L_1 and C_1 per unit length) of the equivalent transmission line. The characteristic impedance $Z_0 = \sqrt{(L_1/C_1)} = \sqrt{(\mu/\epsilon)}$ in the loss-free case, needs a non-zero inductance to give it a non-zero value. Likewise the velocity of propagation $v = \sqrt{(1/L_1C_1)} = \sqrt{(1/\mu\epsilon)}$ needs a non-zero inductance to give a finite propagation velocity, a requirement the authors state in their second paragraph. In trying to dispense with this inductance, the authors' analysis in the Appendix becomes confused. In the equation $T = (1-k/n)^n$ it is certainly true to say that, as n increases indefinitely, T tends to the limit exp $(-k)$ *but only if k is fixed.* Since the authors have put $k = 2nZ_0/R$ the analysis is immediately suspect and this suspicion is confirmed when n then reappears as a finite number.

In spite of appearances, such as introducing ideas of reflections on a transmission line, the authors' analysis is a quasistatic one and the equation they are deriving is quasistatic also. This being so, it is of little consequence whether one assumes an infinite propagation velocity, which the authors object to, or a zero propagation time which the authors are actually doing. Starting from equation 7 (or 6) and ignoring what follows enables us to redefine T as the time for a double pass: the time for n two-way passes is therefore $t = nT$. Also $t = 2l/v = 2l\sqrt{L_1C_1}$ (where l is the length of the line) and we can therefore write the characteristic impedance as $Z_0 = T/2lC_1 = T/2C$ where C is the "total capacitance". Equation 7 now becomes

$$V(t) = V\left[1 - \left(\frac{1-T/2RC}{1+T/2RC}\right)^{t/T}\right]$$

where inductance does not appear explicitly, but it is there implicitly since it controls the value of T: if L_1 becomes vanishingly small, so indeed must T. Saying, as the authors do, that L_1 does not exist is tantamount to saying that T is zero. All that is now necessary is for the limit to be taken as T vanishes. To do this, put $n = t/T$, eliminate T and let $n \to \infty$: this gives

$$V(t) = V[1 - \exp(-t/RC)]$$

All this has not involved, explicitly, a displacement current simply because the analysis has been conducted in circuit terms rather than field terms. This, of course is quite valid provided that only the dominant mode is involved, which it is because the analysis is quasistatic: indeed a circuit analysis is better suited to this type of solution. A field analysis would have *needed* a displacement current, but it would also have needed a magnetic field. Whether the authors like displacement current or not, their quarrel is not with it but with the magnetic field in the dielectric of the capacitor. The analysis in their Appendix actually gets rid of the magnetic field by declaring that the inductance does not exist: it does *not* get rid of the displacement current (in spite of what the title says) since they use capacitance, its circuit equivalent.

The authors' arguments in the second paragraph do not reveal any flaw in the model presented by Maxwell's equations which need displacement current for consistency. This "mathematical manipulation", no matter how convenient it is from that point of view, would have no value whatsoever if the consequence of a proper application of Maxwell's equations was to produce a result that did not conform with the true state of affairs. The authors have not shown this to be the case in their example, and though I consider Maxwell's equations to be postulates that may be approximations to a more general theory (like Newton's laws are), I do not consider their example to present the sufficiently severe test necessary to the nature of the approximations in those fundamental equations. The flaw, as I see it, is in the authors' understanding rather than in the model, as illustrated by the statements in the second paragraph of the article.

Take first the word "suddenly". Why do we have any need to imagine a "sudden" distribution of charge over the plate any more than we need to explain how a charge moves suddenly from the far end of the wire connected to the capacitor plate to the plate itself? As the authors state, there is a finite velocity by which effects are propagated, but why do the authors imagine (if I understand their statement) that displacement current makes this velocity infinite? It is easy to show that *ignoring* displacement current indicates an infinite velocity, and it could be said that this is a valid reason for including it, not getting rid of it.

Now take the statement that the charge on the plates is uniformly distributed. This is not true if the quantities concerned are varying with time. It is a perfectly general principle that uniform fields varying with time cannot exist: we may be able to produce an approximately uniform field even up to fairly high frequencies but it does not invalidate the principle. The uniformly distributed charge is a quasistatic approximation arising from the small reactive effect of the magnetic field within the dielectric on the electric field distribution. In circuit terms this is equivalent to saying that the effect of inductance is negligible. Maxwell's equations certainly do *not* indicate a uniformly-distributed charge, but maybe this is what the authors imply when they say there is a flaw in the model. But by what argument can it be said that the charge distribution *is* uniform? Indeed, we ought to *expect* a non-uniform distribution, and the greater the radius of the plates, the greater will be the non-uniformity in the radial direction, but then the situation is changing from the quasistatic one to a propagation or travelling wave model.

I would suggest that if there is any paradox, the authors have got the wrong culprit. The model they call faulty works fine and the concept of displacement current is essential to it. I agree with the statement "Work on high speed logic design has shown that the model of a lumped capacitance is faulty", but no further. Any lumping must be faulty since it implies no physical size and/or no time delay. In situations where the time delay is important, quasistatic models will need replacing by those that include propagation effects, but this can only be done by including magnetic flux (or its circuit equivalent, inductance) *and* displacement current (or its circuit equivalent capacitance). Declaring that either one or the other has negligible effect gives a quasistatic model (in the loss free case) and clearly this is not what the authors want.

K. O. Sharples,
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The City University,
London, EC1.

The authors reply:

We do not say that (distributed) inductance does not exist in a capacitor. We said that *series inductance* does not exist. The conventional model of a capacitor with stray series inductance is wrong. Thence, the idea of a capacitor's self-resonant frequency is wrong. *Distributed* inductance, such as exists in a transmission line, does exist, and we use the formula $Z_0 = \sqrt{(L/C)}$. We feel that the whole of Mr Sharples's letter founders because he confuses series inductance with distributed inductance.

DISPLACEMENT CURRENT

JUNE 1979

The pattern of magnetic field made when a very sharp edge of voltage propagates along any TEM wave structure is the same as that obtained if the wave front is replaced by a thin sheet of uniform conductor and the current of the wave is applied as a balanced d.c. on one side only of this sheet.

If this experiment is performed it will be found that there is no magnetic field whatever beyond the sheet and no longitudinal magnetic field at any point, despite the fact that lateral current is clearly flowing in the sheet. On page 67 of the March issue this result is described as being absurd, but it is nevertheless true.

Since the field pattern is just the same for the propagating edge as for the d.c. case it seems only reasonable to talk of a "displacement current" when a magnetic field is caused by change of the vector *D* rather than by real current. There is no question whatever of "displacement current" not causing magnetic field in some particular cases, and neither Maxwell nor Heaviside have overlooked a discrepancy in this matter.

K. C. Johnson
Cheadle
Cheshire

The authors reply:

In Mr Johnson's first paragraph, when he writes "uniform conductor" he must of course mean "uniform resistor."

When a TEM signal advances at the speed of light, there is a close mathematical correlation between the *E* field and the *H* field at every point.

When a TEM signal glides through a dielectric edged by a perfect conductor, there is a close mathematical correlation between the *H* field and the electrical current in the surface of the conductor.

D being a mathematical function of *E* and *i* also being a mathematical function of *E*, it is not surprising that the two mathematical derivations from the same source, *E*, correlate, even to the extent that there is a consistent relationship between

$$\frac{d(\epsilon E)}{dt}$$

and i. One could say that these two derivations from E correlate by definition. Since

$$\frac{d(\epsilon E)}{dt}$$

and i are obviously functions of E, it is mathematically impossible for the reverse mathematical process (cf. logs and anti-logs) to produce anything other than the original E field from which i and displacement current are derived.

The key question is, "Does any function which is correctly derived from a real physical entity also have physical reality?" For instance, to carry the point to absurdity, what physical reality can be attached to the "circularity," α, of a circle, defined in terms of the circumference as follows:

$$\alpha = \frac{C^2}{4\pi^{1/2}}$$

from which it can be deduced that the circle's area A is

$$A = \frac{\alpha}{\sqrt{\pi}}$$

We could have just as much futile fun with "circularity" as we do with "displacement current." They are both the results of valid mathematical manipulation. But do they exist physically, and are they useful?

Displacement current has shed no light and produced much fog. Is it anything more than a mathematical derivation from the Poynting Vector, which we call the Heaviside signal?

To put it another way; if we describe an $E \times H$ wave which has an edge, does it have an edge? Displacement current "shows" that we have the thing we defined.

I. Catt, M. F. Davidson, D. S. Walton