Displacement current

— and how to get rid of it

by **I. Catt** and **M. F. Davidson** (CAM Consultants)
and **D. S. Walton** (Icthus Instruments Ltd)

To enable the continuity of electric current to be retained across a capacitor Maxwell proposed a "displacement current". By treating the capacitor as a special kind of transmission line this mathematical convenience is no longer required.

CONVENTIONAL electromagnetic theory proposes that when an electric current flows down a wire into a capacitor it spreads out across the plate, producing an electric charge which in turn leads to an electric field between the capacitor plates. The valuable concept of continuity of electric current is then retained by postulating (after Maxwell)[1] a "displacement current", which is a mathematical manipulation of the electric field E between the capacitor plates which has the dimensions of electric current and completes the flow of "electricity" (Fig. 1 (a) and (b)). This approach permits us to retain Kirchhoff's Laws and other valuable concepts, even though superficially it appears that at the capacitor there is a break in the otherwise continuous flow of electric current.

The flaw in this model is revealed when we notice that the electric current entered the capacitor at one point only on the capacitor plate. We must then explain how the electric charge flowing down the wire suddenly distributes itself uniformly across the whole capacitor plate. We know that this cannot happen since charge cannot flow out across the plate at a velocity in excess of the velocity of light. This paradoxical situation is brought about by a fundamental flaw in the basic model. Work on high speed logic design[2] has shown that the model of a lumped capacitance is faulty, and "displacement current" is an artefact of this faulty model.

The true model is quite different. Electric current enters the capacitor through a wire and then spreads out across the plate of the capacitor in the same way as ripples flow out from a stone dropped into a pond. If we consider only one pie-shaped wedge of the capacitor, as in Fig 1 (c), we can recognise it as a parallel plate transmission line whose only unusual feature is that the line width is increasing (and hence the impedance is decreasing). The

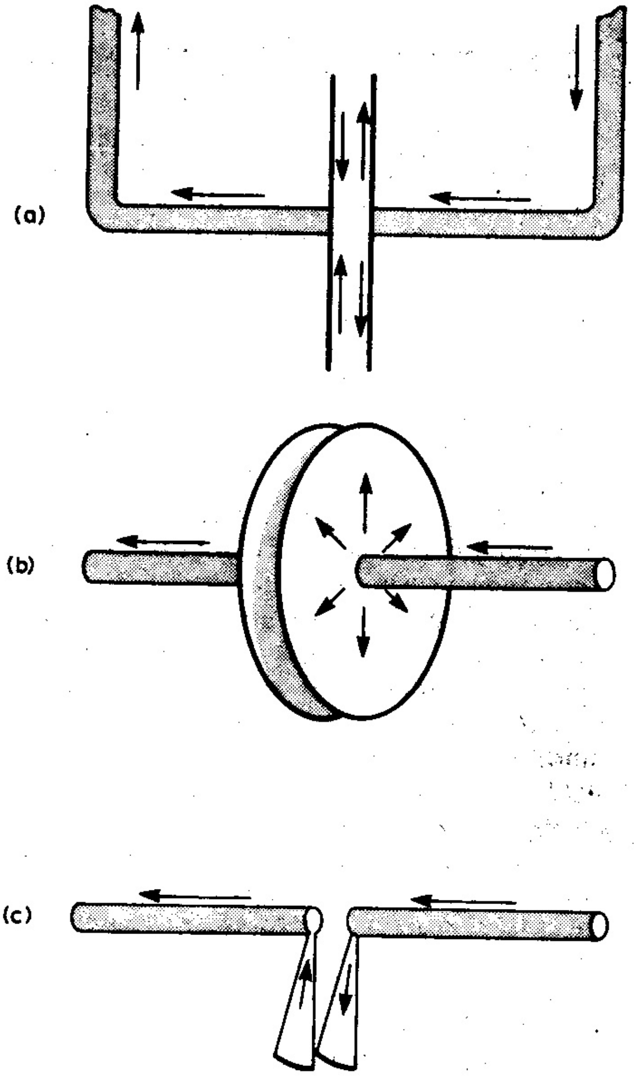


Fig. 1 *Process of current flowing into a capacitor and spreading out across a plate is shown in (a) and (b). The structure in (b) can be considered as being made up of a number of pie-shaped wedges as in (c), each of which is a transmission line.*

capacitor is made up of a number of these pie-shaped transmission lines in parallel, so the proper model for a capacitor is a transmission line.

Equivalent series resistance for a capacitor is the initial characteristic impedance of this transmission line at a radius equal to the radius of the input wires. Series inductance does not exist. *Pace* the many documented values for series inductance in a capacitor, this confirms experience that when the so-called series inductance of a capacitor is measured it turns out to be no more than the series inductance of the wires connected to the capacitor. No mechanism has ever been proposed for an internal series inductance in a capacitor.

Since any capacitor has now become a transmission line, it is no more necessary to postulate "displacement current" in a capacitor than it is necessary to do so for a transmission line. The excision of "displacement current" from Electromagnetic Theory has been based on arguments which are independent of the classic dispute over whether the electric current causes the electromagnetic field or vice versa.

Appendix

Comparison of the transmission line model with the lumped model of a capacitor in an RC circuit.

Taking the above discussion further, consider a transmission line as shown in Fig. 2, assumed to be terminated with a resistance R_T (not shown). The reflectian coefficient is $\rho=(R_T-Z_o)/(R_T+Z_o)$ where Z_o is the characteristic impedance of the line. If the line is open-circuit at the right-hand end, as shown (and therefore R_T is infinite), the $\rho=+1$. We will assume that $R \gg Z_o$.

When switch S is closed (at time $t=0$) a step of voltage $V.Z_0/(R+Z_o)$ is propagated down the line. This reflects from the open circuit at the right hand end to give a total voltage $2V.Z_o/(R+Z_o)$. Reflection from the left end makes a further contribution of $[V.Z_0/(R+Z_0)] \times [(R-Z_0)/(R+Z_0)]$ and so on. In general after n two-way passes the voltage after n passes is V_n and,

$$V_{n+1}=V_n+2\cdot\frac{VZ_0}{R+Z_0}\left[\frac{R-Z_0}{R+Z_0}\right]^n \tag{1}$$

In order to avoid a rather difficult integration it is possible to sum this series to n terms using the formula,

$$=\frac{a(1-\nu^n)}{1-\nu} \tag{2}$$

where a is the first term of a geometrical progression and ν the ratio between terms. (This formula is easily verified by induction.) Substituting in (2) the parameters from (1),

$$\text{i.e. } a=\frac{2VZ_0}{R+Z_0} \tag{3}$$

$$\nu=\frac{R-Z_0}{R+Z_0} \tag{4}$$

We obtain,

$$V_n=\frac{\dfrac{2VZ_0}{R+Z_0}\left[1-\left[\dfrac{R-Z_0}{R+Z_0}\right]^n\right]}{1-\dfrac{R-Z_0}{R+Z_0}} \tag{5}$$

$$=V\left[1-\left[\frac{R-Z_0}{R+Z_0}\right]^n\right] \tag{6}$$

This is the correct description of what is happening as a capacitor charges. We can now go on to show that it is approximated by an exponential. We have

$$V_n=V\left[1-\left[\frac{R-Z_0}{R+Z_0}\right]^n\right] \tag{7}$$

Consider the term,

$$T=\left(\frac{R-Z_0}{R+Z_0}\right)^n$$

$$=\left(\frac{1-Z_0/R}{1+Z_0/R}\right)^n$$

If $Z_0/R \ll 1$ this term is asymptotically equal to

$$\left(1-\frac{2Z_0}{R}\right)^n$$

Now define $k=2Z_0n/R$. Substitution gives:

$$T=\left[1-\frac{k}{n}\right]^n$$

By definition, as $n \rightarrow \infty$ we have,

$$T=e^{-k}=e^{\frac{-2Z_0n}{R}}$$

And therefore:

$$V_n=V\left[1-e^{\frac{-2Z_0n}{R}}\right]$$

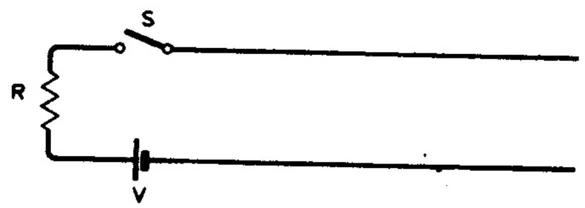
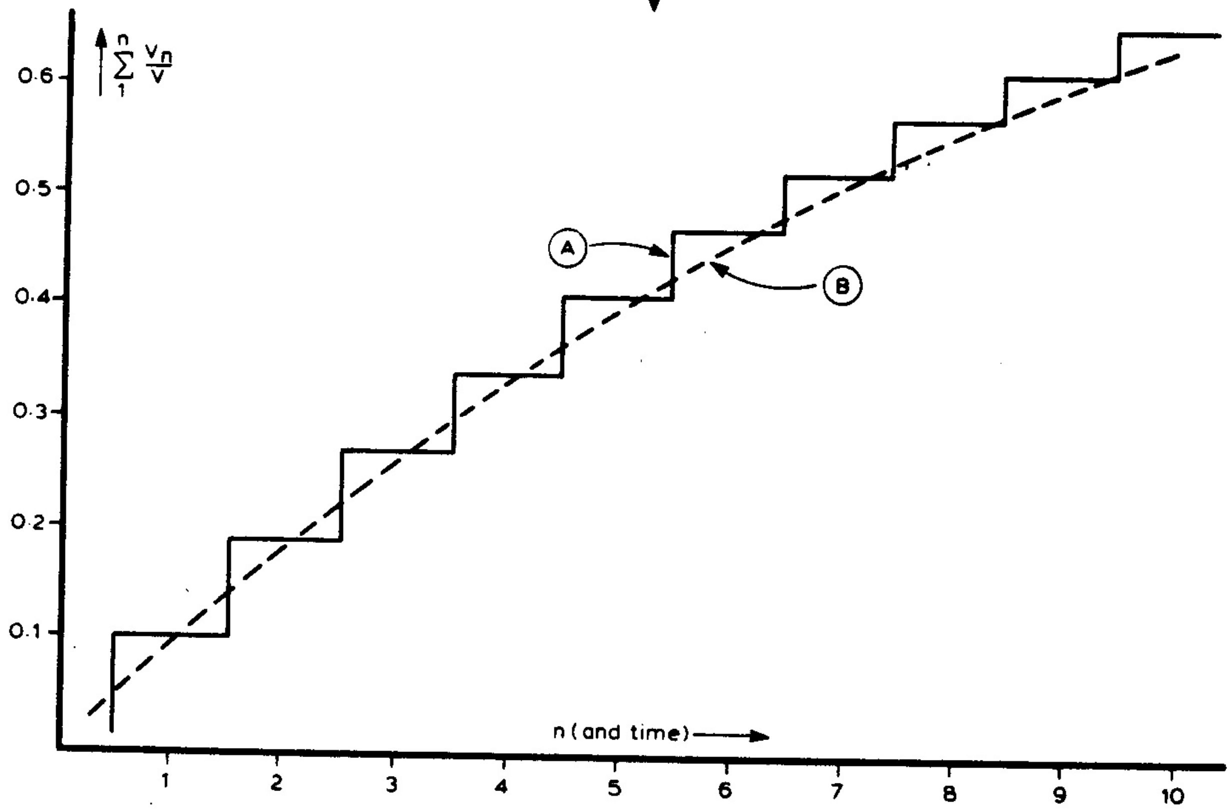


Fig. 2 *An open-ended transmission line.*

Fig. 3 *Comparison of the transmission line model $1-(1-2Z_0/R)^n$ in the curve A with the lumped model $1-e^{-2Z_0n/R}$ in curve B, for $2Z_0/R = 0.1$.*



Now, after time t, $n = V_c t/2l$, where V_c = velocity of propagation.

Therefore

$$V(t) = V\left[1 - e^{\frac{-V_c t}{l} \cdot \frac{Z_0}{R}}\right]$$

For any transmission line it can be shown that:

$$Z_0 = f\sqrt{\frac{\mu}{\epsilon}}$$

$$V_c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$C_l = \epsilon/f$$

where C_l = capacitance per unit length, and f is the same geometrical factor in each case. The "total capacitance" of length l of line $= l.C_l = C$.

$$\text{Hence} \frac{V_c Z_0}{lR} = \frac{1}{RC}$$

and therefore

$$V(t) = V(1 - e^{-t/RC})$$

which is the standard result. This model does not require use of the concept of charge. A graphical comparison of the results is shown in Fig. 3.

References

1. "History of displacement current", I. Catt, M. F. Davidson, D. S. Walton. *Physics Education*, to be published early 1979.

2. "Crosstalk (noise) in digital computers", I. Catt. *IEEE Trans.* EC-16, Dec. 1967, pp. 743-763.

DISPLACEMENT CURRENT

The explanation given by Messrs Catt, Davidson and Walton (December 1978, p.51) of the flow of current 'through' a capacitor without resorting to Maxwell's concept of displacement current is attractive to me, because notwithstanding my immense respect for Maxwell I have always felt that displacement current was a kind of subterfuge to get over a logical difficulty*. But before wholeheartedly accepting this alternative I would like to be given certain reassurances.

At the foot of column 1 the authors point out that the parallel elements of the disk capacitor depicted can be regarded as transmission lines whose characteristic impedance (Z_0) is continuously decreasing towards the far end. So there would be gradual reflection all the way. But in the mathematical proof Z_0 is treated as constant and there is reflection only at the far end. This made me feel I was being conned.

According to Ampère's Law, the connecting leads carrying the charging current must be everywhere encircled by a magnetomotive force numerically equal to the current. In the authors' Fig. 1 the leads are horizontal and the plates are in vertical planes, parallel to one another and also to the n.n.f. around the leads. But what about the m.m.f. in the space between the plates, due to what we have become accustomed to calling displacement current? This current, being a continuation across the capacitor gap of the external circuit current, one naturally sees its m.m.f. also as in a vertical plane. Can the authors show clearly how this follows from the geometry of their transmission line currents, which flow everywhere at right angles to the current in the leads? This aspect is of some importance, since the propagation of radio waves depends on it. Can the authors convincingly get rid of displacement currents in space?

M. G. Scroggie,
Bexhill,
Sussex.

*But I never had, or heard of, a difficulty created by imagining current having to flow across the capacitor plates faster than light. Where did the authors get that idea? And why wouldn't it apply also to the current in the leads?

The authors reply:
The article discusses a circular capacitor. The appendix discusses a rectangular capacitor in order to minimize mathematical complexity. The appendix proves that if a voltage source is switched across a resistor and a rectangular capacitor in series, a waveform results which approximates to an exponential. As Mr Scroggie points out, it does not prove the same for a non-rectangular capacitor.

If you ask us to resolve paradoxes in classical theory, you are asking us to say that we are saying nothing that is fundamentally new; you are asking us not to publish anything. Do you believe that "new" information is only acceptable if it indicates no flaws in the conventional wisdom, i.e. if it is not really new?

As to the m.m.f. in the space between the plates, this have never been measured. If it had been measured it would have been found to be non-uniform, and the revered B. I. Bleaney and B. Bleaney ("Electricity and Magnetism," Clarendon, 1957, p.238) and others would not have written ". . . . the field in between the plates is uniform . . .", which of course it is not; a TEM waveform advancing between the plates of a capacitor (≡ transmission line) creates a field behind itself but not ahead of itself.

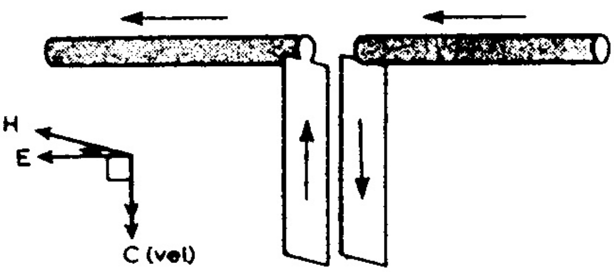
The last paragraph of Mr Scroggie's letter is crucial. If the capacitor were rectangular and oriented much as shown in our Fig. 1(c) then no m.m.f. in the vertical plane would result from current in the capacitor plates. Vertical m.m.f. would mean that the waveform was not TEM, but we know that it *is* TEM and travelling vertically downwards between the capacitor plates. That is, *E* and *H* fields are at right angles to the (downwards) direction of propagation, and therefore are horizontal. This is no more paradoxical than trying to apply Ampère's Law to a TEM step travelling along any transmission line.

Ampère did not know that a TEM wave ($E \times H$) travels forward between two wires at the speed of light. He did not know that a capacitor is a transmission line; he did not know about transmission lines.

These matters will be discussed further in a forthcoming article in *Wireless World*. A paper "The Heaviside Signal" will further clarify the situation (see "Electromagnetic Theory Vol 1," published by C.A.M. Publishing, 17 King Harry Lane, St Albans).

I. Catt, M. F. Davidson, D. S. Walton

Further letters on this subject will be published later. Ed.



(See Electronics and Wireless World, Jan.88, page 54.)

MAXWELL, EINSTEIN AND THE AETHER

I regret that lack of space prevented me from publishing the above forthcoming article in this book.

The summary of the article is as follows;

"To sum up. Einstein says that* relativity, which he believes to have been based on the disappearance of a space with physical properties, is based on Maxwell's Equations, which are now [see pp 187,138] found to contain <u>only</u> information about the physical attributes of that disappearing space.

"By analogy, it would be possible to proclaim a new theory of mechanics which lacked the concept of mass, but which contained both velocity v and momentum mv within it, and which preferably included lots of fancy maths involving momentum and velocity. .... The necessary parameter m, like the rabbit in the hat, could go about its business... firmly hidden in ... a fog of mathematics..."

* See footnote, page 193

C.A.M.