A mathematical rake's progress

Ivor Catt looks back on how he nearly became a maths addict

JANUARY 1986

In my article of last November, I showed that Maxwell's Equations, so long thought to contain the heart and essence of electro magnetism, told us virtually nothing about the subject. Then, in my December article, I discussed the academic mafia's vested interest in knowledge (see panel). Here I try to discover who this group of charlatans*, the maths pushers, are. How does a young student grow up to become part of the social group who live by mathematical nonsense like Maxwell's Equations, and who conspire to prevent the development of a scientific subject in a proper, physical way?

Concern about this question led me to look back on my own education. What pressures were exerted on me to become a mathematical rake?

My experience indicates that the slide is similar to that of the drug addict — a number of small, apparently innocuous, slips downward, culminating in total separation from reality. As we progress through school and college, we are fed a series of potions, each more heady than the last.

The process started with the calculus. My introduction to it, at the age of 15, was worrying and disorienting. It was part of the great disaster which I thought had overtaken me in my first few months in the sixth form. Whereas I had always been good at maths, I found the first few months in the sixth form confusing. Even though Sam Richardson was a very good teacher, and I had help from my mother, a brilliant mathematician, at home, I couldn't understand the basis of what we were learning in mathematics, particularly the calculus.

*The Shorter Oxford English Dictionary entry for this word is particularly apt:

> **Charlatan** 1. A mountebank who descants volubly in the street; esp. an itinerant vendor of drugs, etc. . . .
> 2. An empiric who pretends to wonderful knowledge or secrets . . ., a quack.

However, if we then look up the entry for Empiric, the whole picture backfires on us.

This was a new experience for me. Previously, I had always found maths easy, and scored high marks. Now, suddenly, it was different. This was serious because if I tried to retreat from maths into some other field, all nearby subjects were based on maths anyway. There seemed to be no escape from my new-found inadequacy in mathematics. As the first half-year exams approached I became more and more worried, because still I couldn't grasp the basis of what I was being taught.

The flaw in the calculus package is what I now recognise as the reductionist fallacy a misconception which underlies and undermines western philosophy.* The error is to think that 'the whole is the sum of the parts', no more; that lots of bits of string are quite as useful as (and the same thing as) a long piece of string. Putting it another way, the problem of discontinuities was ignored. I was right to worry.

A whole array of misleading, damaging concepts slipped in with i, or j as we electrical engineers call it. "Two for the price of one"; if $a + jb = c + jd$, then $a = c$ and $b = d$; so we can do two jobs at once. Pretty, but a delusion, similar to the illusion that we can drive better after drinking, and for the same reason — our vision is blurred.

*Titus, H.H., Living Issues in Philosophy. American Book Company, 1964, pp.148, 527, 540 etc.

Hot on the tail of j came that awful array of cons under the appropriate descriptor 'sine'. I shall not develop this theme fully, but only repeat that one FRS went so far as to say that "Physical reality is composed of sine waves". In fact, the sinusoidal wave, which is a camouflaged circle, is Ptolemy's pure, circular epicycles fighting back against Kepler's less pure, more real, ellipse. Kepler, who himself loved the idea of the 'harmony of the spheres', saw a more pure 'equal areas in equal time' rather than a distinctly un-heavenly, earthy, (we would say 'real',) ellipse.

The *Wireless World* July 1981 editorial, 'The decline of the philosophical spirit', contrasts the nineteenth century, when scientists were interested in and capable of distinguishing between the physically real and the mere mathematical construct, and today, when scientists no longer know or care about the difference, and have even developed a philosophy of science which confuses them.*

An example of the destructive effect of sine is the way in which it suddenly appears, unannounced and without justification, on the second page of a text book discussion of the t.em. wave.

In the event, my first half-year exams in the sixth form didn't seem too hard, and I felt that I must have scored over 50%,

*Popper, K. Conjectures and Refutations, R.K.P., 1963, p.100

which would give me a breathing space in which to re-plan my future. To my astonishment, I learned that I had scored 99% and 92%.

However much I might *think* I didn't understand what was going on in maths, the marks I scored 'proved' otherwise. My high scores told me that I *was* still good at maths, as I had always been. However, a nagging suspicion remained with me that something was amiss. I doubted whether I could really have misjudged the situation so badly. Today, I believe that I was correctly judging the situation, and it was my exam marks that were wrong. I was being brainwashed into the belief that understanding was unnecessary, even impossible; that success meant the ability to manipulate the symbolism of the subject, not to understand it. I was being encouraged, the initial carrot being high exam marks, to turn the handle of the mathematical barrel organ, and not to ask too many awkward questions.

I seemed to learn my lesson, and later on, when taking A-levels, I gained a State Scholarship in maths although only 17 years old. This was a remarkable achievement, and should have secured my loyalty to the administrators of the mathematical myth. However, I was already questioning the usefulness of some of this maths, particularly the interminable geometry (since dropped) in the Cambridge Open exam, and so at Cambridge I decided to leave my strong subject, maths, and read engineering.* My background must have made me particularly sceptical. My mother had scooped the lot, gaining the top 'first' in maths in London University, but the payoff to her in benefits in later years proved minimal.

The next piece of blatant brainwashing occurred during my engineering course in Cambridge. We had a lot of thermodynamics, which was very mathematical. One day I asked my tutor, Professor Binnie, what practical interpretation I could place upon an equation containing a college of terms involving the three e's — energy, enthalpy and entropy. His answer was that I should not bother to look for a physical interpretation, but should merely regard it as a piece of algebra to be manipulated according to the rules of algebra. I was shocked by this, and I remain shocked today. Had I left maths and taken up engineering for nothing?

Whereas drawing, or draughting, was strong in the Cambridge Engineering Faculty and seemed to occupy a large part of our time, being the only subject you were not allowed to fail, electricity was weak, rating only one lecture a week, or at

*I love the Heaviside remark; "Whether good mathematicians, when they die, go to Cambridge, I do not know." — Heaviside, O., Electromagnetic Theory, Vol. 3, Dover, 1950. (First published 1903.)

most two. One suspects that conservative Cambridge of the 1950s hoped that this new-fangled electricity thing would prove to be a flash in the pan, and go away soon. (Gaslight, I have been told, was very pleasant; much softer on the eye than electric light.) I suspect that my later success in elecromagnetic theory resulted from the lack of teaching in it that I had sustained while at college.

We did not cover the Laplace Transform, and this set me apart from upstart graduates from red-brick universities, who enjoyed discovering how backward Cambridge was. I was lucky in this omission, because I now feel that transforming is one of the destructive mathematical techniques in engineering that increases the divorce from reality, and which is the legacy to engineers from mathematicians. Whereas to me it is obvious from first principles that to get constant current through a capacitor* you need a continually increasing voltage, I recently found that for a student of Laplace this is the conclusion of a lengthy piece of complex calculation.

Thus was the stage set for Maxwell's Equations, that phoney apology for electromagnetic theory, which held sway for a century and so befogged the subject.

There is a similarity between the maths pushers and drug pushers. Both entice the victim with promises of Elysium. Both gradually increase the dose. In both cases, there is nothing at the end of the rainbow.

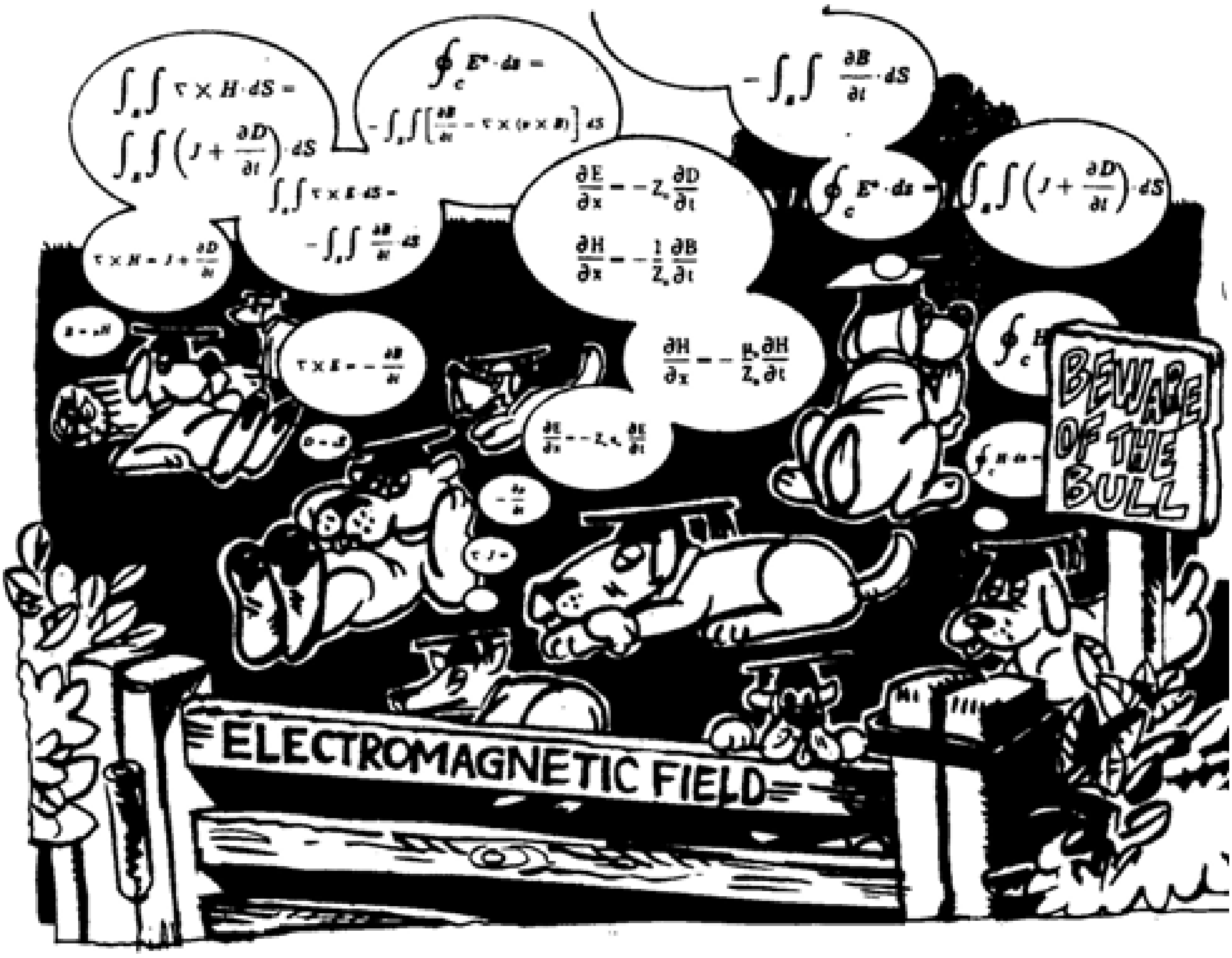
* using theory N

Mathematical mafia

The twisting of historical fact in the hands of the academic mafia is beautifully illustrated by the case of the discovery of the electromagentic theory of light. Obviously, a mathematician would like us to believe that the proposal that light was electromagnetic in nature resulted from subtle manipulations of his electromagentic equations by Professor Maxwell the mathematician. In fact, Whittaker[1] says that the proposal that light is electromagnetic came from Faraday in 1851, when Maxwell was 20. Now it might be asserted that the vague suggestion by Faraday was confirmed and strengthened by Maxwell's mathematics. However, Chalmers[2] says that there is an error in Maxwell's calculations, which "led Pierre Duhem to accuse Maxwell of adjusting his calculation so that he could arrive at a theory of light which he [or should we say Faraday?] already had in mind."

The truth appears to be that the idea preceded the maths; the maths was force-fitted onto the idea, like the ugly sister's shoe; and then the mafia claimed the maths generated the idea. The prince was not hoodwinked, and neither should we be. This racket, of forcing mathematical liturgy onto a reluctant discipline, constantly recurs in science, perhaps reaching its most grotesque in so-called 'computer science' courses.

1. E.T. Whittaker, A History of the Theories of Aether and Electricity, Nelson, 1951, p.194.
2. Chalmers, A.F., Maxwell and the displacement current, *Physics Education*, vol. 10, 1975, p.45.


$\int_s\int \nabla \times H \cdot dS = \int_s\int \left(J + \frac{\partial D}{\partial t}\right)\cdot dS$
$\oint_c E^* \cdot ds = -\int_s\int\left[\frac{\partial B}{\partial t} - \nabla \times (v \times B)\right] dS$
$-\int_s\int \frac{\partial B}{\partial t}\cdot dS$
$\int_s\int \nabla \times E \cdot dS = -\int_s\int \frac{\partial B}{\partial t}\cdot dS$
$\nabla \times H = J + \frac{\partial D}{\partial t}$
$\frac{\partial E}{\partial x} = -Z_0\frac{\partial D}{\partial t}$
$\frac{\partial H}{\partial x} = -\frac{1}{Z_0}\frac{\partial B}{\partial t}$
$\oint_c E^* \cdot ds = \int_s\int \left(J + \frac{\partial D}{\partial t}\right)\cdot dS$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\frac{\partial H}{\partial x} = -\frac{\mu_0}{Z_0}\frac{\partial H}{\partial t}$
BEWARE OF THE BULL
ELECTROMAGNETIC FIELD

C.A.H.

ENERGY TRANSFER

In a realistic physics all interaction should be treated as a discrete cause-effect process, which is intuitively self-evident from the principles of physical reasoning but it is not realised in present day theorising. Such a discrete process generates a power series as the interaction propagator passes to-and-fro and standard analytical techniques can be used to determine whether the series will converge to a finite limit or not. (The information accumulated in the series coefficients corresponds to the field exchange particles of high energy physics.)

If it tends to an asymptototic equilibrium the power series may be replaced by a more compact function from which the power series can be regenerated. The reversible correspondence between power series and generating function representing it is only valid within the radius of convergence for the iterative process of the cause-effect interaction.

This is the crucial problem for advance in theoretical physics: to develop a non-equilibrium behaviour with mathematically self-consistent formulae using continuous variables. Self consistency means that we *assume* that an equilibrium can exist between the variables (such as in a circuit) then proceed to calculate their would-be values.

For instance; Newtons laws of mechanics and gravitation and Einstein's relativistic modification of them, and de Broglie's 'wave' description for electrons in atoms, the formulae of electromagnetic theory and electronic circuit design. These all correspond to asymptotic equilibrium of a discrete interaction: the conditions of interaction under which these self consistent limiting descriptions are valid (as a calculation short-cut to a solution that exists is never even thought about.

The example I can quote is of feedback in a control system which I derived longhand in *WW* Dec. 1983. A finite time delay was supposed in a feedback control system and its response to the simplest possible stimulus of a unit step was described using discrete element analysis.

The 'operational amplifier' formula was rederived including valid limits for convergence, which is essential to understand the negative-feedback instability (so intuitively obvious) and why it is not predicted by the self-consistent op-amp formula. The self consistently derived op-amp formula is beautifully simple but wrong in principle, since the solution cannot exist (meaning well defined finitely stable behaviour) outside the region of convergence.

(As a matter of interest the Nyquist formula is just a special instance of the Curie-Weiss susceptibility law for a loose — coupled system and fits into a wider scheme of 'phase transition' formulae which includes the relativistic formulae for the limiting speed of light to material motion.)

C.A.M.

Upon similar reasoning concerning what mathematical representations mean, the formulae of relativity and general relativity are calculation short-cuts. In all probability Einstein got the right answers for the wrong reasons. The de Broglie wave formula for equilibrium behaviour of electrons in atoms is a piece of brilliant insight but falls into the same classification of self consistency. A more fundamental mechanics is needed if we are to unify physics.

The Catt anomaly is based upon a misconception about the mathematical formulation of electromagentism; classical electromagnetism and its quantum, relativistic modifications can only be valid as a description for asymptotic equilibrium.

The response of any system (transmission line of interacting particles) to the transient of a step stimulus is a non-equilibrium phenomenon. hence the 'anomaly' vanishes as meaningless when it is recognised as an attempt to apply an equilibrium theory to a non-equilibrium situation. What is needed is a more general non-equilibrium theory from which to derive the equilibrium one as a special case.

E and M displacement current are not confused, as Mr Catt suggests (Letters, April). They refer to two separate aspects of an interaction, the asymptotic regime and the initial transient effects respectively where at the ultimate discrete scale of reality the mathematical sophistication of Fourier analysis is beautiful but useless.

Most worrying of all is the thought that the mathematical formulation of our physical laws (using continuous variables and compacted into generally applicable differential equations) implicity precludes certain phenomena from our reasoning which anyone with an ounce of physical insight would see as intuitively obvious though perhaps no so easy to quantify.

Algebra is just a noise-free information processing system (or medium) and logical manipulations do not add anything that is essentially 'new' beyond that physical insight which was encoded into the original formulation of the theory. As with any computing language if you put rubbish in you will get rubbish out; however, one needs to overcome the psychological barrier of the brazenly sophisticated formulae so spuriously created if we are to challenge the conclusions of the theory with *physical* reasoning to attempt unification.

One needs to discuss mathematics in everyday language to truly understand what a formula (usually in a commutative or asymptotic cause-effect logic) represents and not to float adrift in a sea of assumedly understood.

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ENERGY TRANSFER

The reversed polarity of the Hall effect, referred to by R. Petzeratt in the November issue, is not confirmed to p-type semiconductors. The same phenomenon is found in certain metals, such as zinc and cadmium. The mater was a major anomaly until the late 1920s, when it was accounted for in quantum mechnics.

As regards Catt's views concerning electrical conductivity, he seems to have confused my reference (in the February issue) to electrons entering and leaving the conduction band (from and to the valence band) with the metal strips or "bands" ins triplines. This suggeests lack of appreciation of the physical realities, except for the out-dated "cannonball" electron from classical electromagnetism, that participate in the physical processeds in conduction, such as photons and psi waves. My main point was, and remains, that the velocity of electrons does not form a strong enough foundation from which to make such pronouncements as "the death of electric current" and "electric charge does not exist" (December 1980).

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WHAT'S IN A THEORY?

Our Ivor Catt always makes cheerful light reading, especially when he points out the obviousness of some physical statements which are usually mentioned only with bated breath. Typically, in *E & WW* for November 1985 he reminded us that only ***uniquely electromagnetic*** information contained in the two Maxwell equations relating E and H in vacuum lies in the theoretical artefacts μ_0, ε_0; from them one can extrapolate to derive "the impedance of free space" as $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and "the universal velocity of light" as $C = 1/\sqrt{\mu_0\varepsilon_0}$.

The phenomenon is by no means confined to electromagnetic theory. Some of your readers might care to try the following unconventional derivation in the wave theory of matter:—

A completely general expression for 3-dimensional waves in any linear, continuous medium is provided by

$$\nabla^2\psi + \left(\frac{2\pi x}{u}\right)^2 .\psi = 0. \quad ...(1)$$

where ψ (psi) is "whatever it is that oscillates" in the waves and u,x are the wave velocity and frequency respectively. The last are classed as "unobservables" by the theory, so we will simply eliminate them by means of the wave axiom (self-evident truth) $u = \pi v$, obtaining

$$\nabla^2\psi + \left(\frac{2\pi}{\lambda}\right)^2 .\psi = 0. \quad ...(2)$$

Next, on the apparent evidence of experiment we assume these matter-waves obey the momentum precept $p = mv = h/\lambda$, so that

$$\nabla^2\psi + \left(\frac{2\pi}{h}\right)^2 p^2.\psi = 0. \quad ...(3)$$

Now the momentum $p = mv$ is

directly related to the particle's kinetic energy $K = \frac{1}{2}mv^2$ by the form $p^2 = 2mK$; substitution for p^2 leads inexorably to

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}K.\psi = 0. \quad ...(4)$$

Finally, since the total dynamic energy of any particle is defined as $W = (K + U)$, evidently $K = (W - U)$ and therefore

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(W - U).\psi = 0. \quad ...(5)$$

Here we have derived Schrödinger's Wave Equation, that famous, basic statement of the Wave Mechanics which students may approach only with extremest reverence. Given two further assumptions it leads to Schrödinger's mathematical model of the atom — represented as an infinite set of spherical harmonics — which is said to "explain" the atomic line spectra.

The derivation given above may be unfamiliar, but it is both simple and watertight. Step by numbered step it consists of the following physical assumptions:—

(1) let us suppose that linear matter-waves exist;
(2) Then these matter-waves must obey the wave axiom;
(3) The matter-wavelength of a particle is related in this particular way to its mechanical momentum *(de Broglie)*;
(4) The laws of Newtonian mechanics are to apply; and,
(5) A particle's energy is part kinetic, part potential.

Given at (1) the idea of matter-waves and at (2) the axiom which must follow from that, the *only reference* made to any property of the waves, namely $p = h/\lambda$, occurs at (3). The derivation is in fact complete at this point, the other two steps being added to facilitate its practical application. We note that the derivation, and therefore the wave equation itself, is invariant with respect to any definition of υ, so that "any old waves will do" for the wave theory of matter. That is why a Wave Mechanician will give you an odd look if you ask him about the *frequency* of his matter-waves. Nor is any textbook willing to tell us whether these matter-waves are transverse or longitudinal. The theory doesn't know about such things, and it gets away without caring about them either!

As Mr Catt has pointed out in the case of the electro-magnetic theory, so here in the case of the wave-mechanics, and so also as is well known in the case of special relativity, the key theoretical formulations are found to rest upon two, and only two, particular and radical physical assumptions. The rest is algebra.

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ENERGY TRANSFER

P. L. Taylor (*Wireless World*, p. 15 October 1985) likes the choice between e.m. wave energy transfer through space, as required by the Poynting vector, or through wires, as required by the Slepian vector. There is, in fact, a third choice advocated by Cambridge Professor G. H. Livens. Writing 'On the flux of energy in radiation fields' at p. 313 of his 1926 book 'The Theory of Electricity', published by Cambridge University Press, he argued in favour of an alternative to Poynting's theory. Waves do not need to carry energy at their speed of propagation. Their generation merely adds energy to a common pool of energy in the field medium at the locality of the transmitter and their absorption draws on that pool in the locality of the receiver. I like this third alternative, because it is easier for me to picture creation a a big splash in an existing smooth pool of energy than as a big bang appearing from nowhere in a complete void.

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MAXWELL

The consistence of Ivor Catt's misrepresentation of Maxwell's laws is remarkable. Whatever deficiency they do contain, if any, it is certainly not at the elementary level claimed in the issue of November 1985. ('The Hidden Message in Maxwell's Equations').

The basic problem remains apparent ignorance of vectors and the role they play in Maxwellian theory. Ivor Catt seems to think that Maxwell's laws are some kind of elaborate hoax supported by an establishment conspiracy to suppress 'alternative' theories. He also believes that equations (9) and (10) in his latest diatribe represent the views of the conspirators. If they did, he would indeed have a point. But, sad to say, the windmills that this exuberant knight errant is tilting at are significantly different from the reality of Maxwell!

The mistake he makes is fundamental and disastrous. It is entirely necessary to modern em theory that the E field vector is perpendicular to the H vector. Why, then, do equations (9) and (10) not show this? Without the direction property of a vector, em theory would fail to account for such simple phenomena as reflection. Ivor Catt attaches some mystical importance to Z_0; anyone who was properly conversant with EM theory would not. Z_0 is derived from the magnitude of the E and H vectors; their directional property is eliminated and most that is useful in the theory with it. Z_0 is not a 'primitive': it lacks directional information.

Ivor Catt's difficulties with the expression of physical concepts in mathematical form do not seem to be confined to electrical matters. True enough, if he walks along the plank far enough in the direction (hooray for direction!) 'v', 'h' does indeed decrease but so does 'x'. Sorry, Ivor old son, but you are wrong again, as you walk along the plank it is because it is going backwards underneath you that it leads to that sinking feeling.

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C.A.M.

RAKES' PROGRESS

I feel that Mr Catt (January 1986) has missed the purpose of mathematics in science. Mathematics provides a formal description of a theoretical model for empirically discovered results. Such a description is necessary for predicting other results from those already discovered.

Since the mathematical description is of a model, it cannot be expected to fit the world, warts and all. The model will deal with an idealisation; in particular, since the model is not a description of all and everything, it will entail concentrating on some effects and ignoring others. Hence the problems with defining (e.g.) ampères experimentally. Further, formulisations based on "real" numbers entail using limits which are impossible experimentally. Overcoming these problems means inventing a better model, one which includes more forces, or can be described by quantised mathematics.

The universe is quite complex, and it is often useful to deal in idealisations which ignore "irrelevant" forces. Presumably, the next round of "complete" or unified theories will bring together astrophysics, electromagnetism and subatomic physics; however, I suspect that predicting simple effects from such a theory will be very complicated.

It is for precisely the above reasons that it is wrong to attack the use of mathematics in computing science. Here, one is dealing with a situation which is wholly understood, since it has been created synthetically. One can produce complete mathematical descriptions of (e.g.) the languages in use. It is the very artificiality of the computing science world that allows this accurate formalisation, in contrast to the situation generally in physics.

ENERGY TRANSFER

Recently, since Ivor Catt first elaborated his iconoclastic views, several Wireless World readers have surfaced with their own problems relating to electric current theory. To these I would like to add my own.

At school, I was taught that the resistance of a conducting wire was inversely proportional to the cross-sectional area of the wire.

$R \alpha 1/A \alpha 1/r^2$ (where r = radius of the wire) This seemed reasonable since, just as with water running down a pipe, the thicker the wire/pipe, the more space there would be for the charge/water to pass through.

Later, I learned that electric charge only flowed along the surface of a conducting wire, (down to a certain small "skin depth"). This seemed to square with the idea that the residual charge on a conductor is to be found on the surface. However, this would lead one to imply that the resistance would be inversely proportional to the cross-sectional perimeter of the wire.

$R \alpha 1/r$

Furthermore, does a hollow conducting wire (with wall thickness greater than skin depth) have the same resistance as a solid conducting wire of the same radius?

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