<u>On Capacitors as Transmission Lines</u>

Ivor Catt has repeatedly written that there is 'no static field in a capacitor' E.g. in a charged or charging capacitor, there are waves travelling to and fro 'for ever' which do not settle down to a static condition. (for this to make sense, we presumably have to make an exception of a fully discharged capacitor not connected to anything, for which there are no currents or voltages or fields in place – at least not above the molecular level where we may discuss atoms and their structure and movements).

What follows is an attempt to discuss the basis for the claim of no static field.

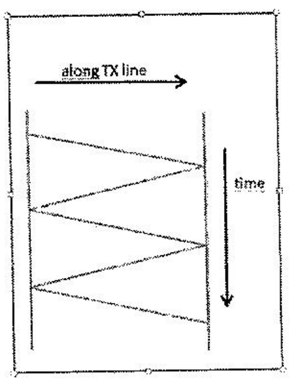
Consider a capacitor connected to an ideal voltage source E, series resistance R, and an ideal switch, initially open and the capacitor discharged. To be more specific, suppose that the resistor is 10kΩ and the capacitor a traditional 0.1µF tubular type comprising two long strips of tinfoil separated by a thin insulating strip, the whole thing rolled up in the usual way for capacitors in 1950s electronics (e.g. about the time when they stopped being called condensers).

The circuit analysis found in elementary physics and engineering text books derives the following equations:

$$Vc = E(1 - \exp\left(-\frac{t}{CR}\right) \quad for \quad t > 0$$

$$Ic = \frac{E}{R}\exp\left(-\frac{t}{CR}\right) \quad for \quad t > 0$$

These equations match what would be measured on a good (analogue, of course) oscilloscope of that era. Field theory and propagation of waves does not arise, because the simple theory involves no space dimensions, since these are not needed for the derivation or analysis. They also correspond, of course, to the usual lumped circuit model of traditional circuit theory, which is accepted as an adequate representation of real behaviour in many situations.



Next, suppose that the capacitor is 'unrolled' so become a long straight strip – two strips of tinfoil separated by one narrow strip of insulator. It is obvious that it then has the nature of an open-circuited transmission line – and if we assume that the resistance of the tinfoil and the conductance of the insulator are negligible, it fits the model of a uniform, linear time-invariant (ULTI) transmission line. Methods for analysing their behaviour are well-known and include the Bewley Lattice diagram method (shown left), or standard derivations and descriptions to be found in many textbooks of the 1950s. (Bewley lattice is also sometimes called a reflection diagram or bouncing diagram)

For the lossless line the characteristic impedance is resistive, given by $Ro = \sqrt{(L/C)}$

So when the switch is closed a voltage step of amplitude $Ro/(R+Ro)$ travels along the line and on reaching the open-circuited end has a 100% reflection back to the source. The current wave step is reflected as minus 100% since the open circuit cannot carry any current (and so 'opposes' the incoming current wave). On reaching the sending end, the voltage step sees a termination of R and so a step is reflected back along the line, and some more energy is dissipated in resistor *R*. At the sending (source) end the size of the reflection back is calculated using the reflection coefficient:

$$\rho_s = \frac{R - Ro}{R + Ro}$$

The wave gets another 100% reflection at the open end, and so the process is repeated, the wave travelling to and fro, getting smaller in amplitude every time it returns to the sending end. The voltage versus time will be a 'stepped version' which closely follows the smooth exponential charging of the elementary text book equations. To get sensible results, it is necessary that $Ro < R$. For the 'unrolled traditional 0.1μF capacitor' charged by the 10kΩ resistor that must be the case, but since the length of the unrolled tinfoil and the thickness of the insulation layer between them are 'unknowns' the actual value can only be guessed at.

Much the same would be seen in the calculations were the RC circuit to be analysed by some standard iterative computer algorithm for solving first order differential equations, e.g. it would provide a stepped approximation to a smooth exponential with the required time-constant.

Now, suppose we model the transmission line as a ladder network of L and C components. This has long been a standard text book approach. Maybe we choose 100 equal inductors and 100 equal capacitors. Analysis would be lengthy but not impossible, and the resulting voltage waveforms would be close to the measured reality. The 'step' would not be vertical and there would be small 'ripples' before and after the step. Using 1000 inductors and capacitors would make the step nearer to vertical and the ripples smaller and of higher frequency but overall the result would be the same. Note that in the 100 inductor and capacitor circuit there is still no dimension of distance so no meaningful discussion or concept of the wave 'travelling along the line' and having a speed in metres/sec. It is still a lumped circuit with no space dimensions, however many Ls and Cs are used.

Now, let's consider the unrolled capacitor as a real transmission line in space. It is well established (e.g. using Poynting Vector, for example) that the energy flowing from the source to the termination travels as an electromagnetic wave in the space outside the line, at right angles to the *E* and *H* fields. Any energy flow in the conductors is inwards and at right angles to the direction of the line. We have assumed that the line is lossless, so the skin depth is zero and there is therefore no energy travelling into the conductors, they simply 'guide' the wave in the space outside. Analysis of the behaviour may be more difficult but will give just the same results as the ULTI line described above. So there will indeed be waves travelling to and fro in support of Catt's claim of 'no static field' in the capacitor being charged up.

<u>Special cases and difficulties</u>

1. It is common practice in the measurement of behaviour of transmission lines in the frequency-domain to be interested in standing waves, etc. and for this purpose a small slot is often cut into the line to allow a probe to go in and 'measure' the internal field. Given that the propagating wave should be <u>outside</u> the line, why is the measurement made inside the line?

2. Suppose that the characteristic impedance of the lossless ULTI line (*Ro*) is the same as the source resistance *R*. The voltage 'step' going along to the end of the line has amplitude *E/2*,

When the reflected 'step' waveform of *E/2* is added the incident step the total voltage on the line is *E*. So when the 'step' coming back from the open-circuit termination reaches the source, it sees a matched termination and so is completely absorbed and there is no further reflection and the voltage all along the line stays at *E*. The reflections therefore stop immediately, contradicting the Catt assertion of no static field. The voltage *E* all along the line corresponds to the capacitor being fully charged. So after one 'round trip delay', there is no further change.

3. When *R* and *Ro* are not equal, as the waves go to and fro, the stepped approximation to the smooth exponential charge of the elementary lumped circuit model continues with smaller and smaller steps as time passes. Eventually the size of the steps will become comparable to the size of a molecule. By then something else has to occur and molecular level physics needs to take over. Even with the old assumption that molecules were small solid particles, the reflections with smaller and smaller steps would become unconvincing.

4. The unrolled capacitor obviously has the form of a transmission line, but when it is rolled up to its proper form, it seems to still be a time-invariant linear transmission line but no longer uniform so that a way to analyse its behaviour is no longer obvious (to me). A further problem is that whereas the energy flow in a straight transmission line is clearly in the electromagnetic field outside the line, when it is rolled up there is no longer 'space' outside the line for this electromagnetic field to travel in, and so the simple TEM form is apparently no longer possible and a different explanation of what happens is needed.

<u>A numerical example</u>

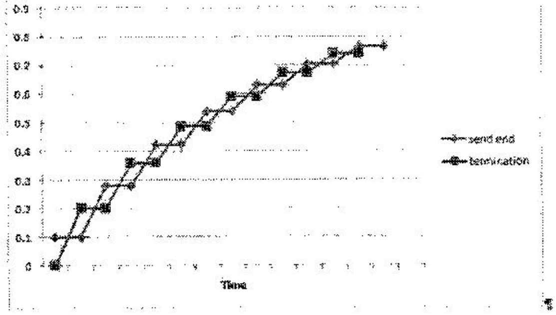
Assume *R = 9Ro* to make the calculations easy. We can simplify more by normalising: assume *E=1* and *Ro=1*. Of course in the real situation, we expect *R >> Ro*

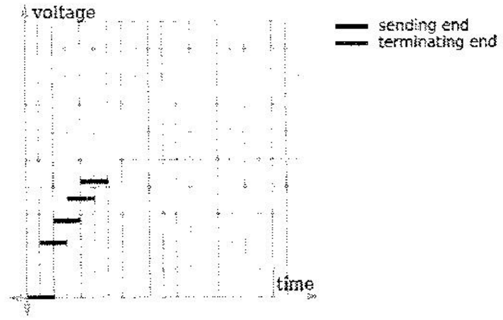
Using the Bewley lattice diagram, for example, it is easy to calculate the voltages at the sending end:

0.1, 0.28, 0.424,0.5992, 0.63146, 0.706188, 0.7631684, 0.8221528…….

and at the open circuit termination the steps occur half way between at:

0, 0.2, 0.36, 0.488, 0.5306, 0.87372, 0.737916, ……





The left hand graph shows the two sequences of values. Note that the sloping links should be vertical, but my graph plotting software does not allow vertical steps. The right hand graph format would be better

Estimating the actual value of *Ro*

The tinfoil strips would be about 20mm wide and the thickness of the insulation very small. Assume that the insulation has μ and ε close to those of free space (so velocity will be near to speed of light).

Any reasonable choices seems to result in a value in the 50Ω to 500 Ω range, so there is no doubt that $Ro \ll R$ and so there will be many more small steps than in the calculation and graph above, so that the stepped approximation will be much closer to the smooth exponential curve of basic circuit theory.

Models, reality and truth

I suggest that it is necessary to be careful not to refer to any of these descriptions as 'truth' – they are all models of reality with varying extents of accuracy depending upon circumstances and details. Deciding about 'truth' is the domain of philosophers, and engineers, scientists and technologists are wise to keep away from that in their work. Humans can use 'reasoning' to reach valid results (but that is really the domain of mathematics, and it is perhaps a matter of just luck that the observed universe seems to behave in a way that mostly does not contradict that mathematical reasoning). The other activity is to make hypotheses, which can be tested against observations, and if the observations conflict with a hypothesis, then the hypothesis has to be rejected. If there is no conflict, that is not sufficient to 'prove truth'. All this might be self-evident but some people act and write as if it were not so.

As a simple example, believers in a flat earth might assume that the reason that the moon wanes from full moon via half moon to no moon and then becomes new moon, then half moon, and eventually full moon once more must be because there is a large strange shaped opaque object in the sky between earth and moon which is moving around and blocking the light from the moon. A perfectly reasonable hypothesis if you assume that the light of the moon is generated by the moon just as the light of the sun is generated by the sun. Of course we now know that really the moon does not generate its own moonlight, it is just acting as a mirror, and this is a much more satisfactory explanation supported by lots more evidence, so that is what most people now accept. However, that is not the full story, because we know that there are influences from other planetary objects, and calculating the orbits of many mutually attracting bodies is not a mathematically soluble problem and can only be approximated. It was problems with the orbit of Mercury (which did not quite fit the established rules) that helped to an acceptance that Einstein's weird ideas might be better (e.g. closer to reality) than Newton's simpler linear ones.

Comments on any of the above welcome at any time !!

Tony Davies

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